SOLUTIONS OF THE EXAMPLES

IN THE

CRAWFORD ALGEBRAS

J. T. CRAWFORD

MACMILLAN

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SOLUTIONS OF THE EXAMPLES

IN THE

CRAWFORD ALGEBRAS

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PREFACE

This Key has been prepared for the use of teachers and private students.

It contains the answers to all the examples to which answers are not found in the text.

With the exception of the oral and elementary examples, it contains solutions of all the examples in the several editions of the Crawford Algebras which are authorized for use in the Secondary Schools of the Provinces of

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J. T. C.

Toronto,
January 1st, 1919.

KEY TO THE CRAWFORD ALGEBRAS

Exercise 1 — Page 1

1. $2 + 2 = 4$. **2.** $5 + 10 + 20 = 35$. **3.** $6 + 4 = 4 + 6$. **4.** $7 \times 8 = 8 \times 7$. **5.** $12 - 5 = 7$. **6.** $10 - 6 = 4$. **7.** $20 - 15 = 5$. **8.** $100 - 30 = 70$. **9.** $36 \div 4 = 9$. **10.** $3 \times 20 + 10 = 70$. **11.** $\frac{1}{2}(7 + 5) = 6$. **12.** $3 + 5 + 7 = 5 + 3 + 7 = 7 + 5 + 3 =$ etc.; $3 \times 5 \times 7 = 5 \times 7 \times 3 = 7 \times 5 \times 3 =$ etc. **13.** $3 \times 4 = 12$, $\therefore 12 \div 3 = 4$. **14.** $4^2 = 16$, $\therefore \sqrt{16} = 4$.

Exercise 2 — Page 4

1. 18, 3, 2, 5, 2, $\frac{1}{3}$, 10. **2.** 8, 2, 15, 21, 1, 3. **3.** 12, 0, 32, 2, 0. **4.** $a + b = 11$, $a - b = 5$, $ab = 24$. **5.** $a + b + c = 22$. **6.** $\frac{x}{y} + c = 13$. **7.** $p + q - r = 12$. **8.** $12 \times 4 - 5 \times 5 + 6 \times 4 - 7 \times 5 + 10 = 22$. **9.** $a + b + c = 42$, $\frac{1}{2}(a + b + c) = 21$. **10.** 40 cents, $7x$ cents, bx cents, mn dollars. **11.** 400, $100x$, $100x + y$, $25a + 10b$. **12.** 72, 43, $36a$, $12b$, $12x + y$, $36m + 12n + p$.

13. a is to be multiplied by m and b by n, and the results added. The value $= 3 \times 2 + 6 \times 5 = 36$.

14. The sum of x and y is to be divided by the sum of a and b. The value $= 22 \div 11 = 2$.

15. 6. **16.** 6. **17.** $2l$, $3l$. **18.** $3x$, $2x$, $5x$, $6x$.

19. (1) The difference between the selling price and the cost is equal to the gain. **20.** 28, 26, $n + 1$, $n - 1$. **21.** $n + 2$, $n - 2$. **22.** $x + 5$, $x - 5$. **23.** 16, $10 + m$, 6, $10 - n$. **24.** $x + n$, $x - m$, $2x$. **25.** $p + 18$. **26.** $\frac{a + b}{c} = \frac{6 + 9}{3} = 5$, $a + \frac{b}{c} = 6 + \frac{9}{3} = 9$, $a \cdot \frac{b}{c} = 6 \times \frac{9}{3} = 18$. **27.** $4a - 4b = 16$. **28.** $ab - cd$, $cd - ab$. **29.** 53, 79.

30. (1) $10 \times 3 + 4 \times 4 - 5 \times 5 + 3 \times 0 = 30 + 16 - 25 = 21$.

(2) $5 \times 3 \times 4 + 2 \times 5 \times 0 - 3 \times 3 \times 5 = 60 + 0 - 45 = 15$.

(3) $\frac{1}{3} \times 3 \times 5 + \frac{1}{4} \times 4 \times 5 - \frac{2}{3} \times 3 \times 0 = 5 + 5 - 0 = 10$.

(4) $\frac{6 \times 3 - 2 \times 4 + 3 \times 5 - 0}{2 \times 3 + 4 - 5 + 0} = \frac{25}{5} = 5$.

Exercise 3 — Page 7

1. 5, 7; 2, 3, 7; 3, 5, 5. **2.** 5, x, y; 2, 3, m, n. **3.** $3 \times abc$, $3a \times bc$, $3b \times ac$, $3c \times ab$. **4.** 5, b. **5.** 72, 2500, 80, 180. **6.** 10^2, 2^4, 3^3, 5^4. **7.** $2a$, $3a$, a^2, a^3, a^4. **8.** 36, x^2. **9.** 27, m^3 cu. in. **10.** 16, 8; 8. **11.** $2^3 - 3 \times 2 = 2$. **12.** $11^2 - 2 \times 11 = 99$. **13.** $(3m)^2 - 3m^2 = 900 - 300 = 600$. **14.** $x^2 + y^2 = 136$, $x^2 - y^2 = 64$. **15.** $3 \times 6^2 = 108$, $6^2 + 2 = 38$, $6 + 2^2 = 10$, $6^2 - 2^2 = 32$, $2.6^2 - 3.2^2 = 60$. **16.** $1^3 + 1^2 + 1 = 3$, $2^3 + 2^2 + 2 = 14$, $3^3 + 3^2 + 3 = 39$, $0^3 + 0^2 + 0 = 0$. **17.** $y = 4.2^2 - 7 = 9$, $y = 4.3^2 - 7 = 29$, $y = 4.(2\frac{1}{2})^2 - 7 = 18$. **18.** 15, $6x$, mn, a^2. **19.** $\dfrac{a^2 + b^2 + c^2}{2a + b - c} = \dfrac{9 + 4 + 1}{6 + 2 - 1} = 2$. **20.** When $x = 2$, $x^3 + 26x = 8 + 52 = 60$; $9x^2 + 24 = 36 + 24 = 60$. When $x = 3$, each $= 105$ and when $x = 4$, each $= 168$. **21.** $x^2 + y^2 = 100 + 25 = 125$, $2xy = 100$, difference $= 25$. **22.** $c = 3\frac{1}{7} \times 14 = 44$; $22 = 3\frac{1}{7}d$ or $d = 22 \div 3\frac{1}{7} = 7$. **23.** $A = 3\frac{1}{7} \times 49 = 154$; $A = 3\frac{1}{7} \times 196 = 616$. **24.** $2^4 \cdot 5^3 = 2^3 \cdot 5^3 \cdot 2 = 10^3 \times 2 = 2000$; $25^2 \times 4^3 = 25^2 \times 4^2 \times 4 = 100^2 \times 4 = 40000$; $125 \times 2^5 = 5^3 \times 2^3 \times 2^2 = 10^3 \times 4 = 4000$.

Exercise 4 — Page 10

1. 5, 1, $\frac{1}{2}$. **2.** 7. **3.** $2b$ and $7b$, $5a^2$ and $-4a^2$. **4.** 6, $6b$, $6c$, $6cy$. **5.** $9a$, $7m$, $16a^2$, $9xy$. **6.** $7x^2 = 7 \cdot 2^2 = 28$ or $3x^2 + 4x^2 = 3 \cdot 2^2 + 4 \cdot 2^2 = 12 + 16 = 28$. **7.** $6a + 12b$. **8.** $7m$, $4ab$, $5x + 2a$, $8a + 14b$. **9.** $4x + 5y + z = 4 \times 3 + 5 \times 5 + 10 = 47$. **10.** $2a^2 + 3a - 20 = 2 \times 6 \times 6 + 3 \times 6 - 20 = 70$. **11.** $100 \times 2 + 10 \times 3 + 4 = 234$; $100 \times 9 + 10 \times 5 + 7 = 957$. **14.** $x + 2x + 4x + 8x = 15x$. **15.** $9x = 72$ or $x = 72 \div 9 = 8$. When $x = 8$, $x + 3x + 5x = 8 + 24 + 40 = 72$. **17.** The average $= \frac{1}{3}(10 + 8 + 15) = 11$; $\frac{1}{3}(3x + 7x + 5x) = 5x$.

Exercise 5 — Page 12

1. 1. **2.** 6. **3.** 13. **4.** 3. **5.** 18. **6.** 48. **7.** 19. **8.** 7. **9.** $3x$. **10.** a. **11.** x. **12.** 2. **13.** 4. **14.** 2. **15.** $42x$. **16.** x. **17.** bz. **18.** 2. **19.** $(p + q) + x$. **20.** $m + (x + y)$. **21.** $2(a + b)$. **22.** $a - (m - n)$. **23.** $(p - q) \div (m + n)$.

24. $8 \times 10 - (2 \times 3 + 2) - 5(10 - 3) = 80 - 8 - 35 = 37$.

25. $7(10 - 3 - 2) - 3(10 - 6 + 2) = 35 - 18 = 17$.

26. $(30 + 6 - 2)(10 - 9) = 34 \times 1 = 34$.

27. $100 + 9 + 4 - 2(30 + 6 + 20) = 113 - 112 = 1$.

28. $\dfrac{10 + 9 - 2}{20 - 15 + 4} - \dfrac{20 - 9 - 3}{10 + 3 - 4} = \dfrac{17}{9} - \dfrac{8}{9} = 1$.

29. $\frac{7}{6} + \frac{1}{12} - \frac{12}{16} = \frac{7}{6} + \frac{1}{12} - \frac{3}{4} = \frac{1}{2}$.

30. The first side $= 5 \times 3 + 3 \times 9 = 42$. The second $= 2 \times 21 = 42$.

Exercise 6 — Page 13

1. Four times the number, one half the no., the square of the no., three times the square of the no. **2.** $5a$, ya. **3.** $x+y$, $y-x$. **4.** $(x+y)-a$. **5.** $10x$, $\frac{1}{10}x$. **6.** $\frac{1}{3}y$. **7.** $\$(x-y)$. **8.** $\$\frac{a}{b}$, $\$\frac{ac}{b}$. **9.** $100a+b-c$. **10.** $\$(x-a-b)$. **11.** $6x$. **12.** $10-x$. **13.** $\$12m$. **14.** 20 miles, ab miles. **15.** $\$(ax-bx)$. **16.** $x+15$, $x-15$. **17.** $a^2-b^2=49-9=40$. **18.** 0, 3, 4. **19.** 24, 0, 25, 13. **20.** $\frac{5}{6}$, 1, $\frac{1}{6}$, 0. **21.** 80, 20, 160, 2625. **22.** $17x$. **23.** $4a$. **24.** 11, $4a$, $\frac{1}{4}(a+b+c+d)$. **25.** $x-16$. **26.** $\$(x+40)$. **27.** 6. **28.** 10^3, 2^5, 3^4, 4^3. **29.** $3\cdot 5$, $3\cdot 5\cdot 7$, $3\cdot a\cdot b$, $5\cdot 7\cdot x\cdot x\cdot y$.

37. $(a+b)-(c+d)=35-15=20$, $(a-b)-(c-d)=5-5=0$. Diff. $=20$. $3(a+b)-5(c-d)=80$, $5(a-d)-3(b-c)=60$. Diff. $=20$.

38. First product $=8\times 9\times 2=144$. Second $=6\times 5\times 4=120$. Diff. $=24$.

Exercise 7 — Page 17

1. 7, 8, 8, 7, 22, 50. **2.** 14, 24, 19, 22, 2, 5.

3. When a no. is added to 8, the sum is 32; 25 is 6 more than a no.; when 15 is subtracted from a no., the remainder is 7; when a no. is subtracted from 10, the remainder is 8; when a no. is subtracted from 17, the remainder is 12.

4. 15, 32, $\frac{1}{3}n$, $\frac{3}{5}n$. **5.** 6, 20, 54, 4. **6.** 6, 3. **7.** 15, 5. **8.** 7, 14. **9.** 18, 30. **10.** 7, 3. **11.** 20. **12.** 6. **13.** 3. **14.** 8. **15.** $5\frac{1}{2}$. **16.** 4. **17.** 12. **18.** 10. **19.** $1\frac{1}{2}$. **20.** 2. **21.** 9. **22.** 30. **23.** 9. **24.** 11. **25.** 7.

Exercise 8 — Page 20

1. 5, axiom 4. **2.** 15, ax. 2; 3, ax. 4. **3.** 8, ax. 1, 4. **4.** 10, 20; ax. 1, 3. **5.** 2, 10; ax. 4, 3. **6.** 9. **7.** 12. **8.** $3\frac{1}{5}$. **9.** $2\cdot 3$. **10.** 5. **11.** 7. **12.** 7. **13.** 4. **14.** 16. **15.** 18. **16.** 10. **17.** 18.

Exercise 9 — Page 20

1. Subtract 11 from each side, then $3x+11-11=47-11$ or $3x=36$. Divide each side by 3, then $x=12$.

3. Add 5 to each side, then $4x-5+5=51+5$ or $4x=56$. Divide each side by 4, then $x=14$.

5. Subtract x from each, then $4x-x=x+21-x$ or $3x=21$ or $x=7$.

8. Subtract 5 from each, then $\frac{1}{2}x + 5 - 5 = 50 - 5$ or $\frac{1}{2}x = 45$. Multiply each by 2, then $x = 90$.

9. Subtract $6x$ from each, then $42 = 3x$ or $x = 14$.

10. Subtract 3 from each, then $10x + 3 - 3 = 3x + 66 - 3$ or $10x = 3x + 63$. Subtract $3x$, then $7x = 63$ or $x = 9$.

12. Subtract 20 from each, then $10x = 0$ or $x = 0$.

13. Add $4m$ to each, then $12m = 36$ or $m = 3$.

15. Add 652 to each, then $12x = 7x + 1080$. Subtract $7x$ from each, then $5x = 1080$ or $x = 216$.

16. Add 9 to each, then $764x = 680x + 21$. Subtract $680x$, then $84x = 21$ or $x = \frac{1}{4}$.

17. $9w + 20 = 50 + 10$ or $9w = 40$ or $w = 4\frac{4}{9}$.

18. If $17x - 11 = 5x + 121$, then $12x = 132$ or $x = 11$.

19. $11y + 60 = 20y - 30$. Add 30 to each, then $11y + 90 = 20y$. Subtract $11y$ from each, then $90 = 9y$ or $y = 10$.

Exercise 10 — Page 21

1. $33 + x = 50$. **2.** $90 - x = 40$. **3.** $2x = 36$. **4.** $5x = 45$. **5.** $2x + 3 = 25$. **6.** $x + 27 = 2x$. **7.** $x - 20 = \frac{1}{2}x$. **8.** $\frac{3}{4}x - 8 = 7$. **9.** 17, 50, 18, 9, 11, 27, 40, 20.

Exercise 11 — Page 23

1. If x is the no., then $x + 37 = 53$ or $x = 16$.

2. If x is the no., then $x - 27 = 5$ or $x = 32$.

3. If x is the no., then $2x + 27 = 73$ or $x = 23$.

4. If x is the no., then $7x - 25 = 59$ or $x = 12$.

5. If x is the no., then $5x + 6 = 2x + 15$ or $x = 3$.

6. If x is the no., then $3x - 36 = 2x$ or $x = 36$.

7. If x is the no., then $x + 19 = 2x + 7$ or $x = 12$.

8. If x is the no., then $5x + 19 = 9x - 41$ or $x = 15$.

9. If x is the less, then $x + 11 =$ the greater, $x + x + 11 = 51$ or $x = 20$.

10. Let x and $x + 15$ be the nos., then $x + x + 15 = 47$ or $x = 16$.

11. If B's is \$$x$ and A's is \$$3x$, then $3x - x = 1500$ or $x = 750$.

12. If the horse cost \$$x$ and the carriage \$$2x$, then $x + 2x = 360$.

13. If the parts are x and $x + 27$, then $x + x + 27 = 93$ or $x = 33$.

14. If the width is x ft. and length $3x$ ft., then $2x + 6x = 72$.

15. If their ages are x and $2x$ years, then $x + 10 + 2x + 10 = 41$.

16. If B gets \$$x$, then A gets \$$(2x + 20)$, then $x + 2x + 20 = 500$.

17. $x + x + 1 = 59$ or $x = 29$. The nos. are 29, 30.

18. $x + x + 1 + x + 2 = 150$ or $x = 49$.

19. If B is x yr., then A is $2x$ and C is $2x + 7$, then $x + 2x + 2x + 7 = 67$.

20. If the width is x ft., the length is $x + 10$, then $2x + 2(x + 10) = 68$.

21. If A gets $\$x$, B gets $\$2x$, and C $\$6x$, then $x + 2x + 6x = 468$.

22. $9\frac{1}{4}m = 333$ or $m = 36$.

23. Let x be the longer, then $20 - x =$ the shorter, then $x = 2(20 - x) + \frac{1}{2}$ or $x = 13\frac{1}{2}$ and the parts are $13\frac{1}{2}$ in. and $6\frac{1}{2}$ in.

24. $5x + 6 = 3x + 40$ or $x = 17$.

25. If the sum is $\$x$, then $\frac{1}{20}x = 48$ or $x = 960$.

26. If the cost is $\$x$, then $\frac{9}{10}x = 2.61$ or $x = 2.90$.

27. If A gets $\$x$, B gets $\$3x$, C gets $\$(4x + 100)$, then $x + 3x + 4x + 100 = 1496$ or $x = 174.50$, so that A gets \$174.50, B \$523.50, C \$798.

28. If B has $\$x$ and A $\$5x$, then $5x - 63 = 2x$ or $x = 21$.

29. $\frac{1}{2}x - 20 = \frac{1}{3}x + 10$ or $x = 180$.

30. If the first sold x, the third $4x$, the second $2x$, $x + 4x + 2x = 42$.

31. $\frac{1}{3}x + \frac{1}{2}x = 55$ or $\frac{5}{6}x = 55$ or $x = 66$.

32. If the payments were x, $2x$, $4x$, $8x$, $x + 2x + 4x + 8x = 4500$.

33. $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 52$ or $\frac{13}{12}x = 52$ or $x = 48$.

34. If the parts are x, $2x$, $3x$, then $x + 2x + 3x = 72$.

35. $x + \frac{1}{2}x - \frac{1}{3}x = 98$ or $\frac{7}{6}x = 98$ or $x = 84$.

36. (1) $a = 200$, (2) $b = 3$, (3) $c = 12$.

Exercise 12 — Page 25

1. See Art. 15, page 18. **2.** When $x = 18$, the first side $= 3 \times 18 - 7 = 47$. The second side $= 2 \times 18 + 11 = 47$. **3.** The first side $= 3(8 + 6) = 42$. The second $= 5(8 - 1) = 35$, so that 8 is not a root. **4.** (*a*) $3x = 6$, $x = 2$. (*b*) $4x = 8$, $x = 2$. (*c*) $5x = 15$, $x = 3$. (*d*) $10 = 2x$, $x = 5$. (*e*) $6x = 2$, $x = \frac{1}{3}$. **5.** \$14000, \$2800. **6.** \$230. **7.** 810, 270, 1620. **8.** 6. **9.** 44, 264; 14, $1\frac{3}{4}$. **10.** 256, 1610. **11.** 56. **12.** \$4. **13.** 30. **14.** \$371. **15.** 80, 200, 250, 600. **16.** 16, 6, 16. **17.** 1000 sq. ft. **18.** 12. **19.** $6\frac{1}{2}$. **20.** 500. **21.** Find a number which is 20 less than three times the number. **22.** \$50, \$60, \$80. **23.** 80 cents. **24.** 35. **25.** \$2040, \$2160, \$2200. **26.** \$250. **27.** 3, $\frac{1}{2}$, 6, 1. **28.** The first side $= 2 \times 5 \times 8 = 80$. The second $= 4 \times 9 \times 1 + 4 \times 11 = 80$. **29.** 50800 sq. mi. **30.** $6500 = 26x$, $x = 250$; $\frac{13}{12}x = 3380$, $x = 3120$; $31 = 155x$, $x = \frac{1}{5}$.

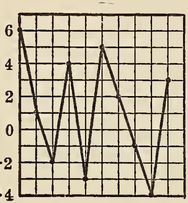
Exercise 13 — Page 28

1. 60°, 59°, 54°. **2.** 6 and 10. **3.** 62°, 52°. **4.** 10°. **5.** 12 and 2, 8 and 12. **6.** About 1.5° ; 1°. **7.** 1 and 3.30 ; 12 P.M. and about 4.20 ; about 5.30 and 10.30. **8.** 11 and 12 ; 4 and 5. **9, 10.** See diagrams on page 11, Figs. 1, 2.

Exercise 14 — Page 29

1. + 35°, + 25°, − 5°, + 15°, − 15°, + 5°, + 30°. **2.** Tuesday and Thursday. **3.** 40°, 35°. **4.** 10°, − 40°. **5.** 76°. **6.** See Fig. 3 on page 11.

Exercise 15 — Page 33



2. \$10, − \$20, \$10, − \$10, \$10, − \$10. **3.** 16°. **4.** − 10°. **5.** 20° South. **6.** 150 *m.* **7.** \$120. **8.** That the top of the tree was below the window ; that the bottom of the well was below sea level. **9.** − \$20, − 20 %. **10.** 30 *m.* **11.** 20°. **12.** $a - b$, $b - a$. **13.** 100, 95, 28, 180, 400, − 100. **14.** 45. **15.** 23. **16.** − 2. **17.** \$$(a - b)$, an overdraft of \$10. **18.** A will have \$50 − \$10 + \$40 = \$80 and B \$20 + \$10 − \$40 = − \$10. **19.** − 27 lb. **20.** See diagram on this page.

Exercise 16 — Page 34

1. − \$5, − \$16, + \$2. **2.** 43°. **3.** 7 lb., − 4 lb., − 9 lb., − 6 lb., 0 lb., *a* lb. **4.** 18, − 14. **5.** 38°. **6.** A starts 3 yd. ahead of the mark and B 3 yd. behind. A has 97 yd. to run and B 103 yd. **7.** It means 10 yr. before the present time. **8.** 4°. **9.** − 20 min. **10.** − 5. **11.** 646. **12.** See Fig. 4 on page 11. **13.** $-3\frac{1}{2}$. **14.** See Fig. 5 on page 11. **15.** + 1 · 5°, + 2°, + 3 · 5°, + 2 · 5°, + 5 · 5°, + 3°, + 1°, − .5°, − 1°, − 1 · 5°. **16.** See Fig. 6 on page 11.

Exercise 18 — Page 38

1. 8 ft. 5 in. **2.** $8x + 5y$. **3.** $7a - 14b$. **4.** 13 hr. 37 min. 34 sec. **5.** $12a - 8b + 6c$. **6.** $-9a - 6b + 12c$.

7.
$$\begin{array}{r} a + b - c \\ a + 2b - 3c \\ \underline{5a + 3b - 11c} \\ 7a + 6b - 15c \end{array}$$

8.
$$\begin{array}{r} 5x^2 - 7x + 6 \\ x^2 - 5x + 3 \\ \underline{4x^2 - 2x } \\ 10x^2 - 14x + 9 \end{array}$$

11.
$$\begin{array}{r} a - 2b + c - 3d \\ 2a - 5b + c - d \\ \underline{a - b + c - d} \\ 4a - 8b + 3c - 5d \end{array}$$

GRAPHICAL SOLUTIONS

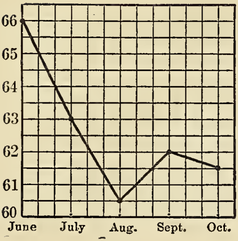


FIG. 1

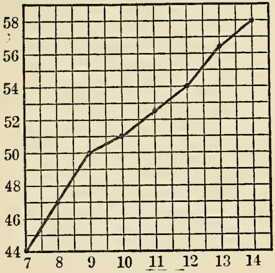


FIG. 2

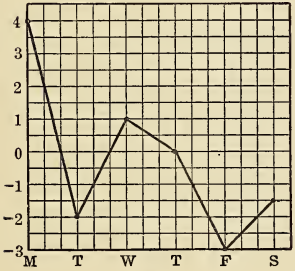


FIG. 3

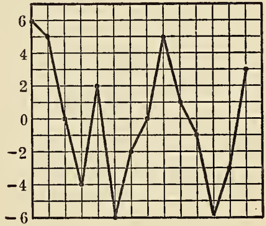


FIG. 4

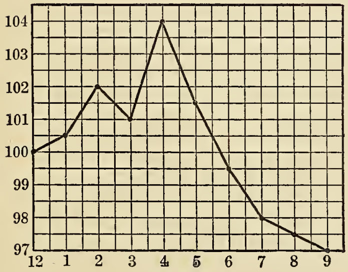


FIG. 5

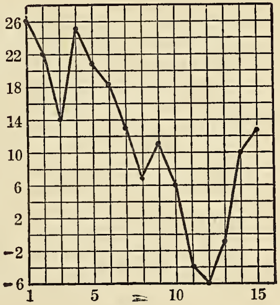


FIG. 6

12.
$$\begin{array}{r}5x-3y\\ -2y+z\\ 3x-y+4z\\ \hline 8x-6y+5z\end{array}$$

14.
$$\begin{array}{r}a^2+2b^2-3c^2\\ 2a^2+5b^2-c^2\\ 3a^2+b^2-2c^2\\ \hline 6a^2+8b^2-6c^2\end{array}$$

16.
$$\begin{array}{r}\frac{1}{3}x+\frac{1}{4}y-\frac{1}{5}z\\ \frac{2}{3}x+\frac{3}{4}y-\frac{2}{5}z\\ x+\phantom{\frac{1}{4}}y-\frac{2}{5}z\\ \hline 2x+2y-\phantom{\frac{2}{5}}z\end{array}$$

Exercise 19 — Page 40

1. 3 ft. **2.** \$17. **3.** -5 lb. **4.** $-10°$. **5.** 4. **6.** $2a^2$.
7. $6x$. **8.** 0. **9.** $8a+b$. **10.** $-x-5y$. **11.** $2a$. **12.** $5a$.

13.
$$\begin{array}{r|l}3x+5y & =8\\ 2x-3y & =-1\\ -4x-y & =-5\\ 6x-4y & =2\\ \hline 7x-3y & =4\end{array}$$

15.
$$\begin{array}{r|l}2a+3b-5c & =0\\ 6a-4b+c & =3\\ 3a+2b+4c & =9\\ \hline 11a+b & =12\end{array}$$

16.
$$\begin{array}{r|l}3a-5b & =-2\\ 4b-3c & =1\\ -3a+4c & =1\\ a+b+c & =3\\ \hline a+2c & =3\end{array}$$

17.
$$\begin{array}{r|l}\frac{1}{2}x+\phantom{\frac{1}{2}}y-\frac{1}{2}z & =1\\ \frac{3}{2}x-\frac{1}{2}y+\frac{1}{2}z & =1\frac{1}{2}\\ x+\frac{1}{2}y+\phantom{\frac{1}{2}}z & =2\frac{1}{2}\\ \hline 3x+\phantom{\frac{1}{2}}y+\phantom{\frac{1}{2}}z & =5\end{array}$$

18.
$$\begin{array}{r|l}a-2b+3c & =2\\ b-2c+3d & =2\\ 2a+c-3d & =0\\ -3a+b-2c & =-4\\ \hline 0 & =0\end{array}$$

19.
$$\begin{array}{r|l}3x+5y-2z & =6\\ 2x-3y+4z & =3\\ 4x+y-5z & =0\\ 6x+2y+3z & =11\\ \hline 15x+5y & =20\end{array}$$

20.
$$\begin{array}{r}3ab-4ac+5bc\\ -2ab+5ac-4bc\\ -ab-ac+3bc\\ \hline 4bc\end{array}$$

21.
$$\begin{array}{r}6a^2-5ab+b^2\\ 3a^2+7ab-2b^2\\ a^2-ab+b^2\\ \hline 10a^2+ab\end{array}$$

22.
$$\begin{array}{r}2x^2-3y^2+4z^2\\ x^2+5y^2-6z^2\\ -3x^2+2y^2+2z^2\\ \hline 4y^2\end{array}$$

24. Adding, $11x+4=48$ or $x=4$. When $x=4$, $13x-7=45$, $2x+5=13$, $6-4x=-10$, and their sum is 48.

25. First sum $=9x-18$, second $=36-9x$. Then $18x=54$.

Exercise 20 — Page 41

1. 1. **2.** 4. **3.** -1. **4.** -5. **5.** a. **6.** $3b$. **7.** $-a$.
8. $-7ab$. **9.** $6x^2$. **10.** $-4p$. **11.** $-11m$. **12.** $-6a$.

16. $2m+3n+5m-n+3m-5n=10m-3n$.

17. $6x+3y-4z+x+2y-z+y+z-7x=6y-4z$.

18. $a-b+b-c+c-a=0$.

19. $x+a-2b+c+b-2c+a+c-2a+b=x$.

21. $(2a-b+5c)+(a-3b-2c)=3a-4b+3c$.

22. $2x+3+3x-5+5x-1=57$ or $10x-3=57$, $x=6$.

23. $8x-7-4x-3=-5x-7+7x-2$ or $4x-10=2x-9$, $x=\frac{1}{2}$.

Exercise 22 — Page 44

1. 38. **2.** -14. **3.** $-2a$. **4.** $24x$. **5.** $4a^2$. **6.** $-3b$. **7.** $7x^2y$. **8.** $-17m^2$. **9.** $a+9b$. **10.** $8x-7y$. **11.** $8n$. **12.** $3x^2-3x+5$. **13.** $2a-2b-2c$. **14.** $-2x^2+9x+1$.

Exercise 23 — Page 44

1. 11. **2.** -13. **3.** -6. **4.** $-9a$. **5.** $-5x$. **6.** $-5m$. **7.** $-15abc$. **8.** -8. **9.** $2x+6y$. **10.** $-a-5b$. **11.** $4x^2+2$. **12.** $9m-9n$. **13.** $3m-7n+9p$.

14. $(-4a+7b-6c)+(6a-4b-5c)=2a+3b-11c$.

20. First remainder $=3a+6b+c$, second $=2a+9b-3c$, third $=2b$. **21.** First sum $=5p+4q-9r$, second $=4p+4q+4r$. **23.** 5, 10, 7, $3b$, x^2+6x-5. **24.** The excess $=a^2-a-7$. **25.** The sum $=-a-b-c$. **26.** The sum $=-2x+4$.

Exercise 24 — Page 46

1. $10+3=13$. **2.** $-5+6=1$. **3.** $-7a+4a=-3a$. **4.** $8x+3x=11x$. **5.** $-2m+3m=m$. **6.** $+b+b=2b$. **7.** $8+4+2=14$. **8.** $8ab-10ab+7ab=5ab$. **9.** $m+3m+5m=9m$. **10.** $-4x^2-3x^2+7x^2=0$. **11.** $5x-2y-2x+4y=3x+2y$. **12.** $3a-11b-5a+8b=-2a-3b$. **13.** $2a-3b+5c-a+4b-5c=a+b$. **14.** $a+b+2a-3b-4a+3b=b-a$.

15. $a+b-c-b-c+a+a+b-c=3a+b-3c$.

16. $6x^2-3x+5+2x^2-5x-6-5x^2+8x-2=3x^2-3$.

18. $2a+3b-c=2\times 1+3\times 2-(-3)=2+6+3=11$.

19. $a-(b-c)=a-b+c=20-10+7=17$ or $a-(b-c)=20-(10-7)$.

20. (1) $2x-3-x+4=8$ or $x+1=8$ or $x=7$.
(2) $3x-1-x+3-x-7=40$ or $x-5=40$ or $x=45$.
(3) $1-4+x-5+x-6+x=52$ or $3x-14=52$ or $x=22$.

21. $5x-6-(3x-11)=70$ or $5x-6-3x+11=70$ or $x=32\frac{1}{2}$.

23. (1) $a+3b-11c-b-3c+8a-c-5a+2b=4a+4b-15c$.
(3) $-3x+y-2z-2x+3y-4z-3y+6z+5x=y$.
(4) $-a+b-b+c-c+d-d+a=0$.

Exercise 25 — Page 47

2. They are x, $x+1$, $x+2$, $x+3$. Their sum is $4x+6$.

3. $(n-2)+(n-1)+n+(n+1)+(n+2)=5n$.

8. $6a-7b+4c-(3a-5b+6c)=3a-2b-2c$.

10. The coefficients are 6, -11, 1, -3. Their sum is -7.

11. $0-(x-y)=0-x+y=y-x$.

13. $2a-b-c-(a-b+c)=a-2c$ or $a-b+c-(2a-b-c)=2c-a$.

18. (2) $x-(y-z)=x-y+z$
$=a+2b-3c-(b+2c-3a)+(c+2b-3a)=a+3b-4c$.

19. The first sum is $.8x+.2y$, the second is $.6x+.2y$, the difference is $.2x$.

20. $3x^2-5x+11-(3x^2-8x+17)=3x-6$. When $x=0$, there is no difference and they are equal. When $x=1$, $3x-6=-3$, and the second is really the greater.

23. $5x-3-x+4-x+2=27$ or $3x+3=27$, $x=8$.

24. $3x-2-(x-7)=63$ or $2x+5=63$ or $x=29$.

25. The sum $=6x+4a-3b+2c=6x+4-6+6=6x+4$.

26. $(a+b)-(c+d)=a+b-c-d$
$=2x+3+5-3x-(3x-\frac{1}{2})-(\frac{3}{2}-6x)=2x+1$.

27. $2-x+\frac{1}{2}-\frac{1}{4}+3x=16\frac{1}{4}$ or $2x+2\frac{1}{4}=16\frac{1}{4}$, $x=7$.

28. $0-(2m-7n-4x)=-2m+7n+4x$.

29. $a-1+b-1+c=a+b+c-2$; $1-(a+b+c-2)=3-a-b-c$.

30. The sum $=3a+b-5c$. The excess $=3a-2b$.

Exercise 26 — Page 50

1. $6xy$. **2.** $20mn$. **3.** $2xy$. **4.** $12x^2$. **5.** $2x^2$. **6.** $12abxy$. **7.** a^3. **8.** $6y^4$. **9.** a^2bc. **10.** $8x^5$. **11.** $20p^6$. **12.** $15x^3y$. **13.** $16x^4$. **14.** $6a^2x^2$. **15.** t^9. **16.** a^3bc. **17.** $30a^3$. **18.** $64a^3$. **19.** $\frac{1}{2}abc$. **20.** $36b^4$. **21.** $20m^2n^2$.

Exercise 27 — Page 51

1. -42. **2.** -12. **3.** 20. **4.** -3. **5.** $-6x$. **6.** $-xy$. **7.** $2xy$. **8.** $-6mn$. **9.** $3x^2y$. **10.** ab. **11.** 36. **12.** axy. **13.** $2mv^2$. **14.** $6x^2$. **15.** $6x^4$. **16.** $-x^3$. **17.** $6a^3$. **18.** $3abcd$. **19.** $-10x^3y$. **20.** a^4. **21.** $-3a^3b^2$. **22.** $-5x^4y^5$. **23.** $-3a^3b^5c^5$. **24.** $-5x^5y^3z^4$.

Exercise 28 — Page 52

1. -60. **2.** 60. **3.** -60. **4.** $-abc$. **5.** $-24a^3$. **6.** 2. **7.** $6x^2y^2$. **8.** -27. **9.** -64. **10.** 120. **11.** 36. **12.** $-8x^3$. **13.** $-120a^5$. **14.** $24x^2y^2$. **15.** $-60x^3y^3$. **16.** -1. **17.** $4a^2$, $9x^2y^2$, $16a^4b^2c^2$. **18.** -125, $-x^3$, $-8x^6$.

20. $3(-2)^2+2(-2)-5=3\times 4-4-5=3.$

22. $(-2)^2+(-3)^2+(-4)^2=4+9+16=29;\quad (-2-3-4)^2=(-9)^2=81.$

23. $(6-4)^2+(4-2)^2+(2-6)^2=2^2+2^2+(-4)^2=4+4+16=24.$

24. $a^3+b^3+c^3=8+1+(-27)=-18.\quad 3\,abc=3\times 2\times 1\times(-3)=-18.$

25. $(x-y)^3-(x^3-y^3)=5^3-3^3+(-2)^3=125-27-8=90.$

26. $3(-1)+2(-2)+(-3)-4(-4)=-3-4-3+16=6.$

27. $(-1)^2+(-2)^2+(-3)^2+(-4)^2=1+4+9+16=30.$

28. $(-1)(-2)+(-1)(-3)+(-2)(-3)+(-3)(-4)=2+3+6+12=23.$

29. $(-1)^2(-4)^2-(-2)^2(-3)^2=1\times 16-4\times 9=-20.$

30. $(-1)(-2)(-3)+(-2)(-3)(-4)+(-3)(-4)(-1)+(-4)(-1)(-2)=-6-24-12-8=-50.$

31. $(-1)^3+(-2)^3+(-3)^3+(-4)^3=-1-8-27-64=-100.$

Exercise 29 — Page 53

1. $8\,a+4\,b.$ **2.** $21\,a-14\,b.$ **3.** $-12\,m+30\,n.$ **4.** $8\,x^2-6\,xy.$ **5.** $-6\,xy+8\,y^2.$ **6.** $-2\,a^2-5\,ab+ac.$ **7.** $6\,x-22.$ **8.** $15\,x-6\,y.$ **9.** $-6\,x+2\,y.$ **10.** $3\,x^3+15\,x^2-6\,x.$ **11.** $10\,x^3y-5\,x^2y^2.$ **12.** $15\,mp-3\,m^2p^2.$

13. $3\,a+3\,b+4\,b+4\,c+5\,c+5\,a=8\,a+7\,b+9\,c.$

14. $2\,x-4\,y+3\,x-3\,y-4\,x+3\,y=x-4\,y.$

15. $6\,m-9\,n-5\,m+5\,n+2\,m+4\,n=3\,m.$

16. $4\,a-8\,b+4\,c-3\,b+6\,c-3\,a-10\,c+8\,a+10\,b=9\,a-b.$

17. $a-\frac{3}{2}\,b+\frac{1}{2}\,a+\frac{5}{4}\,b+\frac{5}{2}\,a+\frac{1}{2}\,b=4\,a+\frac{1}{4}\,b.$

18. $x^2-x+2\,x^2-6\,x+3\,x^2+15\,x=6\,x^2+8\,x.$

19. $a^3-a^2+a+3\,a^2+3\,a-6-2\,a^2-4\,a+6=a^3.$

20. $3\,x^3-6\,x^2+6\,x-6\,x^3-8\,x^2+10x+4\,x^3+5\,x^2-6\,x=x^3-9\,x^2+10\,x.$

21. $-2\,ab+2\,ac-2\,ad-3\,ac+3\,ad-3\,ab-ad+ab+ac=-4\,ab.$

22. 11. **23.** 28. **24.** 1. **25.** 7. **26.** 5.

Exercise 30 — Page 55

1.
$$\begin{array}{r} x+3 \\ x+4 \\ \hline x^2+3\,x \qquad \\ 4\,x+12 \\ \hline x^2+7\,x+12 \end{array}$$

2.
$$\begin{array}{r} 2\,x+7 \\ x+1 \\ \hline 2\,x^2+7\,x \qquad \\ 2\,x+7 \\ \hline 2\,x^2+9\,x+7 \end{array}$$

3.
$$\begin{array}{r} x+5 \\ 2\,x+2 \\ \hline 2\,x^2+10\,x \qquad \\ 2\,x+10 \\ \hline 2\,x^2+12\,x+10 \end{array}$$

4.
$$\begin{array}{r} 3x+4 \\ 2x+3 \\ \hline 6x^2+8x \\ 9x+12 \\ \hline 6x^2+17x+12 \end{array}$$

5.
$$\begin{array}{r} a-3 \\ a-4 \\ \hline a^2-3a \\ -4a+12 \\ \hline a^2-7a+12 \end{array}$$

6.
$$\begin{array}{r} a-5 \\ a+3 \\ \hline a^2-5a \\ +3a-15 \\ \hline a^2-2a-15 \end{array}$$

7.
$$\begin{array}{r} b-4 \\ 2b-3 \\ \hline 2b^2-8b \\ -3b+12 \\ \hline 2b^2-11b+12 \end{array}$$

8.
$$\begin{array}{r} 3a-5 \\ 2a+5 \\ \hline 6a^2-10a \\ 15a-25 \\ \hline 6a^2+5a-25 \end{array}$$

9.
$$\begin{array}{r} 2x-3 \\ 2x+3 \\ \hline 4x^2-6x \\ 6x-9 \\ \hline 4x^2 \qquad -9 \end{array}$$

10.
$$\begin{array}{r} x+y \\ x-y \\ \hline x^2+xy \\ -xy-y^2 \\ \hline x^2 \qquad -y^2 \end{array}$$

11.
$$\begin{array}{r} 2x-3a \\ 5x-a \\ \hline 10x^2-15ax \\ -2ax+3a^2 \\ \hline 10x^2-17ax+3a^2 \end{array}$$

12.
$$\begin{array}{r} 3a-7c \\ 3a+7c \\ \hline 9a^2-21ac \\ 21ac-49c^2 \\ \hline 9a^2 \qquad -49c^2 \end{array}$$

13. $6a^2-7ab-20b^2$. **14.** $2x^2-3xy-35y^2$.

15. $2ac+2bc-ad-bd$. **16.** $3ac-9bc-4ad+12bd$.

17. $x^2-2xy+y^2$.

18. $4a^2-4ab+b^2$, $4a^2-12ab+9b^2$, $16a^2+40a+25$, $9a^2+24ab+16b^2$.

19. $x^2+3x+2+x^2+x-6=2x^2+4x-4$.

20. $3(a^2-4)+2(a^2-4a-5)=3a^2-12+2a^2-8a-10=5a^2-8a-22$.

21. It is correct since $(2x-3)(4x+5)=8x^2-2x-15$.

22. Each side $=12x^2-34x+24$.

23. $mx+my=m(x+y)=2\cdot 14(43\cdot 7+56\cdot 3)=2\cdot 14\times 100=214$.

26. $2(2a^2-ab-b^2)-3(a^2-ab-2b^2)=4a^2-2ab-2b^2-3a^2+3ab+6b^2$.

27. $(x+3)(x+4)-(x+2)(x-9)=x^2+7x+12-x^2+7x+18=14x+30$.

28. $3x+9-2x-8=x+1$; $2x-10-x+3=x-7$; $(x+1)(x-7)=x^2-6x-7$.

30. $x^2-11x+10=x^2-11x+24$. This is an impossible equation.

31. $(6x^2+13x+6)+(6x^2-13x+6)=12x^2+12$; $(12x^2+25x+12)+(12x^2-25x+12)=24x^2+24$. The diff. $=12x^2+12$.

32. $(x^2-6x+9)+(x^2-4)+(x^2+6x+5)=3x^2+10$.

33. $(3a+5)(2a-2)-(2a-5)(3a+2)=(6a^2+4a-10)-(6a^2-11a-10)=15a$.

34. $(x^2+2x+1)+(x^2+4x+4)+(x^2+6x+9)=3x^2+12x+14$.

35. $x^2 - 2x - 35 = x^2 - 3x - 10$ or $x = 25$.

36. $x^2 - 2x + 1 = x^2 - 3x - 10$ or $x = -11$.

37. $6x^2 - 5x + 1 = 6x^2 - 8x - 8$ or $3x = -9$, $x = -3$.

38. $x^2 + 9x - 22 = x^2 - 8x + 7 + 107$ or $17x = 136$, $x = 8$.

39. $x^2 + x + x^2 + 3x + 2 = 2x^2 + 8x + 6$ or $-4x = 4$, $x = -1$.

40. $x^2 + 2x + 1 + x^2 + 4x + 4 + x^2 + 6x + 9 = 3x^2 - 12x + 12 + 14$, $x = \frac{1}{2}$.

Exercise 31 — Page 58

1. $3y$. **2.** $5c$. **3.** $8m$. **4.** $5xy$. **5.** $2a$. **6.** $6x^2$. **7.** $6a^2$. **8.** $5m^2$. **9.** $3a^2b$. **10.** $3pq^2$. **11.** $8a^2$. **12.** $48x^2y$. **13.** $3y$. **14.** $2a^2$. **15.** mv. **16.** $5x^2$. **17.** $4a^2$. **18.** $5y^2z^2$. **19.** $2a^2b$.

Exercise 32 — Page 59

1. -4. **2.** 3. **3.** -5. **4.** -1. **5.** 2. **6.** 2. **7.** 0. **8.** 0. **9.** $-3a$. **10.** $-b$. **11.** $-ay$. **12.** 3. **13.** $-5a^3$. **14.** $-2a^2$. **15.** $-9x^2$. **16.** $2m^2$. **17.** $-x^2y^2$. **18.** $2a^2$. **19.** pr. **20.** $-9m^2n$. **21.** $-3 \cdot 2x^2$.

Exercise 33 — Page 60

1. $3a^2 + 2a$. **2.** $6x^2 + 4x - 2$. **3.** $3x - 2$. **4.** $4m^2 - 1$. **5.** $x + y$. **6.** $-6a + 2b$. **7.** $x - y$. **8.** $a^3 + a - 1$. **9.** $3x - 2y$. **10.** $2b + 2$. **11.** $-3a^2 + 4a - 2$. **12.** $ab - a^2$. **13.** $a^2 + 2a$. **14.** $2 - 5x + 3x^2$. **15.** $6y^2 - 4y$. **16.** $x + 2 + 3x - 3 = 4x - 1$. **17.** $b + c + c + a + a + b = 2(a + b + c)$. **18.** $a + 3 - a - 2 = 1$. **19.** $(x^2 - 4 + x^2 - 6x + 8) \div 2 = (2x^2 - 6x + 4) \div 2 = x^2 - 3x + 2$. **20.** $x + y + y - x = 2y$. **21.** $10a \div 2a = 5$. **22.** $c - b - c + a = a - b$. **23.** $a^2 - a + a - 1 + 1 = a^2$. **24.** $(2x^2 + 8x - 24) - (x^2 - 5x - 24) = x^2 + 13x$. **25.** $x - 10 + x + 5 + 2x - 3 = 40$ or $4x - 8 = 40$, $x = 12$.

Exercise 34 — Page 60

9. First product $= 6x^2 - 13x + 6$, second $= 6x^2 + 13x + 6$.

10. First product $= 10x^2 - xy - 3y^2$, second $= 6x^2 - 13xy + 6y^2$.

13. $(2m - 3n)^2 + (3m - 2n)^2 = 4m^2 - 12mn + 9n^2 + 9m^2 - 12mn + 4n^2$

14. It is true since $(5x - 6y)(3x + 2y) = 15x^2 - 8xy - 12y^2$.

15. $(a - b)(a + b) = a^2 - b^2$, $(a^2 - b^2)(a^2 + b^2) = a^4 - b^4$.

16. $x^2 - 2x + 3 + 3x^2 + 2x - 3 = 4x^2$.

17. $6x^2 + 13x + 6 = 6x^2 + 17x - 3$ or $9 = 4x$, $x = 2\frac{1}{4}$.

18. $2a^2 - ab - 3b^2 + 3a^2 - 2ab - b^2 = 5a^2 - 3ab - 4b^2$.

19. $x^2 + 12x + 27 = x^2 + 11x + 30$ when $x = 3$. $x^2 + 12x + 27$ cannot equal $x^2 + 12x + 32$ for any value of x.

20. $a^2 - 2a + 1 + a^2 - 4a + 4 + a^2 - 6a + 9 = 3a^2 - 12a + 14$.

21. $(3x - 4y)(4x + 3y) - (2x - 3y)(3x + 2y) = 12x^2 - 7xy - 12y^2 - 6x^2 + 5xy + 6y^2 = 6x^2 - 2xy - 6y^2$.

22. $2a^2 + 1 + 3 - a - 2a^2 = 4 - a$.

23. $2(-3)^2 + 3(-3) - 1 = 18 - 9 - 1 = 8$; $2(-4)^2 + 3(-4) - 1 = 19$.

25. $2x + 3y = 2a^2 - 6a + 2 + 6a^2 - 3a - 3 = 8a^2 - 9a - 1$.
$4x - 2y = 4a^2 - 12a + 4 - 4a^2 + 2a + 2 = -10a + 6$.
$\frac{1}{a}(x + y) = \frac{1}{a}(3a^2 - 4a) = 3a - 4$.

26. $\frac{ax - by}{2} = \frac{2a^2 + ab - ab - 2b^2}{2} = \frac{2a^2 - 2b^2}{2} = a^2 - b^2$.
$\frac{4x - 2y}{3a} = \frac{8a + 4b - 2a - 4b}{3a} = \frac{6a}{3a} = 2$.
$\frac{x^2 - y^2}{3} = \frac{4a^2 + 4ab + b^2 - a^2 - 4ab - 4b^2}{3} = a^2 - b^2$.

27. $(6b - 4c - 2b + 3c)(9b - 6c - 4b + 6c) = (4b - c)5b = 20b^2 - 5bc$.

28. $2^3 + 2^3 + (-4)^3 - 3 \cdot 2 \cdot 2(-4) = 8 + 8 - 64 + 48 = 0$.

Exercise 35 — Page 63

1. On simplifying, the statement becomes $8x + 24 = 8x + 24$ and is an identity.

2. It becomes $3x^2 + 21x = 3x^2 + 11x + 10$ and is an equation.

3. It becomes $x^2 - 6x + 4 = x^2 - 6x + 4$ and is an identity.

4. It becomes $3x^2 - 19x + 26 = 3x^2 - 23x + 54$ and is an equation.

5. It becomes $x^3 + ax^2 + a^2x + a^3 = x^3 + ax^2 + a^2x + a^3$ and is an identity.

6. It becomes $x^2 - x - 6 = x^2 + 8x - 3$ and is an equation.

Exercise 36 — Page 65

1. $x = 6$. To verify, first side $= 24 - 4 = 20$, second side $= 12 + 8 = 20$.

2. $x = 3$. First side $= 9 - 7 = 2$. Second side $= 8 - 6 = 2$.

3. $x = 3$. First side $= 3 - 9 = -6$. Second side $= 9 - 15 = -6$.

4. $x = 30$. First side $= 2 \times 25 = 50$. Second side $= 30 + 20 = 50$.

5. $y = 11$. First side $= 5 \times 9 = 45$. Second side $= 3 \times 15 = 45$.

6. $x = 7$. **7.** $x = 10$. **8.** $x = 1\frac{1}{4}$. **9.** $x = 5\frac{1}{2}$. **10.** $x = 8$. **11.** $x = 4$. **12.** $x = 7$. **13.** $x = 5$. **14.** $x = 18$. **15.** $x = 1$. **16.** $x = 1$.

17. $x = 5$. **18**. $x = 10$. **19**. $x = 2$. **20**. $x = 3$. **21**. $m = -\frac{1}{2}$. **22**. $h = -4$. **23**. $x^2 + 10x + 25 - (x^2 + 6x + 9) = 40$, $\therefore 4x + 16 = 40$, $\therefore x = 6$. **24**. $x^2 + 10x + 25 - (16 - 8x + x^2) = 21x$, $\therefore 18x + 9 = 21x$, $\therefore x = 3$. **25**. $y = \frac{1}{2}$. **26**. $x = -28$. **27**. $x = 15$. **28**. $x = 3$. **29**. $2(x^2 - 2x + 1) - 3(x^2 + x - 6) = 32 - (x^2 - 7x + 12)$, $\therefore 2x^2 - 4x + 2 - 3x^2 - 3x + 18 = 32 - x^2 + 7x - 12$, $\therefore -7x + 20 = 7x + 20$, $\therefore x = 0$.

30. Since $10x + 11 = 5x - 9$, $5x = -20$ or $x = -4$.

31. On simplifying it becomes $15x - 26 = 15x - 26$, which is an identity and true for all values of x.

32. Since $5(a - 3) - 3(a - 7) = 28$, $a = 11$.

33. Since $12 + 7x + 4x + 3 + 9 - 5x = 0$, $x = -4$.

34. On substituting $x = 2$, the equation becomes $12 = 6 + k$, $\therefore k = 6$.

35. When $x = 10$, the first side $= 13 \times 14 + 15 \times 16 = 422$.

36. Since $(2x + 7)(6x + 3) - (3x + 2)(4x - 5) = 141$, $\therefore 12x^2 + 48x + 21 - 12x^2 + 7x + 10 = 141$, $\therefore 55x + 31 = 141$, $\therefore x = 2$.

37. Since $(y - 3)(y + 3) - (y + 4)(y - 7) = 40$, $\therefore y^2 - 9 - y^2 + 3y + 28 = 40$, $\therefore y = 7$.

38. $(5 - 3k)(7 - 2k) = (11 - 6k)(3 - k)$, $\therefore 35 - 31k + 6k^2 = 33 - 29k + 6k^2$, $\therefore k = 1$.

39. Since $x^2 - 10x + 25 - (x^2 - 10x + 21)$ is equal to 4, it cannot be equal to 0 for any value of x.

40. Since $(x + 3)^2 = (x - 1)(x + 6)$ or $x^2 + 6x + 9 = x^2 + 5x - 6$, $\therefore x = -15$.

41. Since $3(2x - 1) - 12(x - 3) = 5x - 22$, $x = 5$.

42. $3ax = 6a^2$, $\therefore x = 6a^2 \div 3a = 2a$.

43. Since $Pa = Qb$, $10 \times 12 = 15b$, $\therefore b = 8$.

44. If the weight is w, then $6 \times 85 = 10w$, $\therefore w = 51$.

45. Let x ft. = the distance, then $16x = 8(12 - x)$, $\therefore x = 4$.

46. If $F = 77$, then $C = \frac{5}{9}(77 - 32) = 25$.

47. When $C = 0$, $F = 32$; when $C = 40$, $F = 108$; when $C = 100$, $F = 212$; when $C = -10$, $F = 14$; when $C = -50$, $F = -58$.

48. When $C = F$, $C = \frac{5}{9}(C - 32)$, $\therefore 9C = 5C - 160$, $\therefore C = -40$.

Exercise 37 — Page 68

1. Multiply by 2, then $3x = 2x + 10$, $\therefore x = 10$.

2. Multiply by 6, then $3x = 2x + 12$, $\therefore x = 12$.

3. Multiply by 6, then $3x - 2x = 60$, $\therefore x = 60$.

4. Multiply by 12, then $6x + 4x + 3x = 312$, $\therefore x = 24$.

5. Multiply by 12, then $8x + 9x = 12x + 60$, $\therefore x = 12$.

6. Multiply by 6, then $3x = 4x - 24$, $\therefore x = 24$.

7. Multiply by 6, then $4y = 3y + 27$, $\therefore y = 27$.

8. Multiply by 8, then $x + 2x + 4x = 8x - 32$, $\therefore x = 32$.

9. Multiply by 20, then $10x - 4x = 5x + 20$, $\therefore x = 20$.

10. Multiply by 10, then $15m - 14m = 40$, $\therefore m = 40$.

11. Multiply by 18, then $9x - 6x = 8x - 270$, $\therefore x = 54$.

12. Multiply by 4, then $2x + x = 7 - 4x$, $\therefore x = 1$.

13. Multiply by 20, then $4x + 40 = 30 + x - 4x$, $\therefore x = -1\frac{3}{7}$.

14. Multiply by 4, then $2x - 3 + 28x = 12x + 6$, $\therefore x = \frac{1}{2}$.

15. Multiply by 12, then $4x - 3x = 32$, $\therefore x = 32$.

16. Multiply by 2, then $x - 3 = 40$, $\therefore x = 43$.

17. Multiply by 10, then $14x + 4 = 20x - 5$, $\therefore x = 1\frac{1}{2}$.

18. Multiply by 10, then $x + 1 - 30 = 0$, $\therefore x = 29$.

19. Multiply by 12, then $4x + 3x - 24 = 60$, $\therefore x = 12$.

20. Multiply by 20, then $5x - 5 + 4x + 12 = 160$, $\therefore x = 17$.

21. Multiply by 4, then $2x - 6 + x - 5 = 0$, $\therefore x = 3\frac{2}{3}$.

22. Multiply by 60, then $15x - 90 = 20x + 100 + 12x - 156$, $\therefore x = -2$.

23. Multiply by 60, then $60x - 20 + 25 = 15x + 24x + 12$, $\therefore x = \frac{1}{3}$.

24. Multiply by 105, then $35x + 70 + 210 = 21x + 84 + 15x + 90$, $\therefore x = 106$.

25. Multiply by 40, then $10x - 30 = 16x - 32 + 15x - 25$, $\therefore x = 1\frac{2}{7}$.

26. Multiply by 12, then $4y - 12 - 3y + 15 = 12$, $\therefore y = 9$.

27. Multiply by 20, then $5x + 5 - 4x + 4 = 20$, $\therefore x = 11$.

28. Multiply by 88, then $11x - 77 - 8x + 24 = 0$, $\therefore x = 17\frac{2}{3}$.

29. Multiply by 12, then $6x - 12 - 3x - 6 = 4x - 12$, $\therefore x = -6$.

30. Multiply by 12, then $6x + 6 - 3 = 12x - 8x + 4$, $\therefore x = \frac{1}{2}$.

31. Multiply by 12, then $3x - 10x - 18 = 4x - 18$, $\therefore x = 0$.

32. Multiply by 12, then $72 - 6x + 6 - 4x + 8 = 9 - 3x$, $\therefore x = 11$.

33. $5x - 10 = 3.65$, $\therefore x = 13.65 \div 5 = 2.73$.

34. $2.34 = 4x + 6$, $\therefore 4x = -3.66$, $\therefore x = -.915$.

35. $.5x - .25x - .2x = 3$, $\therefore .05x = 3$, $\therefore x = 60$.

36. $.2x - .2 + .5x - 4.5 = 3$, $\therefore .7x = 7 \cdot 7$, $\therefore x = 11$.

37. Multiply by 616, then $264x - 792 - 56x - 56 = 231x - 1078$, $\therefore x = 10$.

38. Multiply by 12, then $3x + 18 - 3x + 16 - 12 = 2x + 6$, $\therefore x = 8$.

39. Multiply by 60, then $40 - 20x + 45 - 15x + 48 - 12x + 50 - 10x + 45 = 0$, $\therefore x = 4$.

40. Multiply by 1260, then $140x - 140 - 630 + 315x - 180x + 90 + 84 - 126x = 0$, $\therefore x = 4$.

Exercise 38 — Page 71

1. If the no. is x, then $23x + 117 = 232$, $\therefore x = 5$.

2. If the no. is x, then $2x - 7 = 95$, $\therefore x = 51$.

3. If the no. is x, then $235 - 3x = 217$, $\therefore x = 6$.

4. If the no. is x, then $5x + 33 = 7x + 18$, $\therefore x = 7\frac{1}{2}$.

5. If the no. is x, then $\frac{1}{3}x + \frac{1}{4}x = 35$, $\therefore x = 60$.

6. If B has $\$x$, then A has $\$(3x + 10)$, $\therefore x + 3x + 10 = 250$, $\therefore x = 60$.

7. Let x be the greater, then $81 - x$ is the less, $\therefore x - 6(81 - x) = 4$, $\therefore x = 70$.

8. If the no. is x, then $\frac{1}{7}x - \frac{1}{8}x = 2$, $\therefore x = 112$.

9. If the no. is x, then $x - 42 = 59 - x$, $\therefore x = 50\frac{1}{2}$.

10. If the numbers are x, $x + 1$, $x + 2$, then $x + x + 1 + x + 2 = 129$, $\therefore x = 42$.

11. Let the first be x, then the second is $x - 15$ and the third is $x + 21$, $\therefore x + x - 15 + x + 21 = 114$, $\therefore x = 36$.

12. If B has $\$x$, A has $\$(x + 16)$ and C has $\$(x - 8)$, $\therefore x + x + 16 + x - 8 = 176$, $\therefore x = 56$.

13. If the cost is $\$x$, then $\frac{114}{100}x = 2280$, $\therefore x = 2000$.

14. Let the first be x, then the second is $2x$ and the third $3x$,

$$\therefore x + 2x + 3x = 420, \quad \therefore x = 70.$$

15. Since $8x + 5(x + 5) + 3(x + 25) = 2020$, $x = 120$.

16. If x is the no., then $x - 31 = \frac{1}{6}x - 1$, $\therefore x = 36$.

17. If x is the no., then $6x - 35 = 35 - x$, $\therefore x = 10$.

18. If $\$x$ is the price of each cow, then $\$(x - 70)$ is the price of each pig, $\therefore 7x + 17(x - 70) = 754$, $\therefore x = 81$.

19. If x is the no., then $\frac{1}{2}(x - 10) + 40 = x - 30$, $\therefore x = 130$.

20. If the nos. are x and $x + 1$, then $\frac{1}{3}x + \frac{1}{4}(x + 1) = 44$, $\therefore x = 75$.

21. Let x be the greater, then $\frac{1}{7}x + \frac{1}{3}(46 - x) = 10$, $\therefore x = 28$.

22. If x is the greater, then $x = 1\frac{1}{4}(237 - x)$, $\therefore x = 131\frac{2}{3}$.

23. If $\$x$ is the cost, then $\frac{93}{100}x = 110.25$, $\therefore x = 125$. 116.25

24. If they are x and $x + 1$, $(x + 1)^2 - x^2 = 17$. $\therefore x = 8$.

25. Let x be the number of half-dollars, then $30 - x$ is the no. of quarters, $\therefore\ 50x = 25(30 - x)$, $\therefore\ x = 10$.

26. If the time is x years, then $35 + x = 2(7 + x)$, $\therefore\ x = 21$.

27. If it is x years, then $x + 20 = 2(x - 10)$, $\therefore\ x = 40$.

28. If it is x years, then $35 + x = 7 + x + 5 + x$, $\therefore\ x = 23$.

29. If the nos. are $x - 2$, x, $x + 2$, then $\frac{1}{4}(x - 2) + \frac{1}{2}x + \frac{1}{5}(x + 2) = 17$, $\therefore\ x = 18$ and the nos. are 16, 18, 20.

30. If C's share is $\$x$, B's is $\frac{3}{4}x$, and A's is $\frac{4}{5}$ of $\frac{3}{4}x$, then $x + \frac{3}{4}x + \frac{3}{5}x = 705$, $\therefore\ x = 300$.

31. If the sum is $\$x$, then $\frac{2}{100}x + \frac{7}{200} \times 2x = 135$, $\therefore\ x = 1500$.

32. If $\$x$ is the sum, then $\frac{3}{100}x + \frac{4}{100}(x + 50) = 12\frac{1}{2}$, $\therefore\ x = 150$.

33. If there are x quarters and $52 - x$ ten-cent pieces, then $25x + 10(52 - x) = 1000$, $\therefore\ x = 32$.

34. If the room is x ft. square, then the carpet is $(x - 4)$ ft. square, then $x^2 - (x - 4)^2 = 160$, $\therefore\ x = 22$.

35. If $x°$ is the greatest, the others are $x - 35$ and $x - 10$, then $x + x - 35 + x - 10 = 180$, $\therefore\ x = 75$.

36. Let x ft. be the width, then $(x + 4)$ ft. is the length, $\therefore\ (x + 2)(x + 6) = x(x + 4) + 52$, $\therefore\ x = 10$.

37. $\dfrac{m}{4} + \dfrac{m + 2}{3} = \dfrac{2m + 2}{3\frac{1}{2}} + \dfrac{1}{4}$, $\therefore\ m = 13$ and $m + 2 = 15$.

38. If A has $\$x$, then B has $\$(65 - x)$ and C has $\$(95 - x)$, $\therefore\ 65 - x + 95 - x = 100$, $\therefore\ x = 30$.

39. (1) If 10 is subtracted from 5 times a number, the result is 60; find the number. (2) Four times a number is greater than the number by 24; what is the number? (3) The sum of the half and the third of a number is 10 less than the number; find the number. (4) When 5 times a number is subtracted from 23, the remainder is the same as when 4 is subtracted from four times the number; find the number.

40. If he sells x apples, the selling price is $\frac{5}{3}x$ cents and the cost is $\frac{3}{5}x$ cents, $\therefore\ \frac{5}{3}x - \frac{3}{5}x = 128$, $\therefore\ x = 120$.

41. If x is the less, then $\frac{1}{3}x = \frac{1}{5}(147 - x) + 9$, $\therefore\ x = 72$.

42. If his brother has x, then $\frac{4}{5}x - 25 = \frac{3}{4}(x - 25)$, $\therefore\ x = 125$.

Exercise 39 — Page 74

1. Let the numbers be x and y, of which x is the greater. Then $x + y = x - y + 2y$, which is true.

2. To prove $(x + y) - (x - y) = 2y$.

3. To prove $\frac{1}{2}(x + y) + \frac{1}{2}(x - y) = x$.

4. To prove $(x+y)x = x^2 + xy$.

5. To prove $(x+y)^2 = (x-y)^2 + 4xy$.

6. Let them be x, $x+1$, $x+2$, then it is required to prove that $x + x + 1 + x + 2 = 3(x+1)$.

7. Let them be x and $x+2$, then it is required to prove that $2(x+1)^2 = x^2 + (x+2)^2 - 2$.

8. The square of the sum of two numbers increased by the square of their difference is equal to twice the sum of the squares of the numbers.

Exercise 40 — Page 75

3. $x = 6$. 4. An identity. 5. 7. 6. 17. 7. 20, 30.

8. It is an identity.

9. Since $(5-3x)(7-2x) = (11-6x)(3-x)$, $x = 1$.

10. $\dfrac{4x-6}{5} = \dfrac{6x-8}{25} + \dfrac{262}{1000}$. $\therefore x = 2.04$.

11. If each invested $\$x$, then $x + 2600 = 3(x + 200)$, $\therefore x = 1000$.

12. If it holds x gal., then $\frac{7}{8}x - 36 = \frac{1}{2}x$, $\therefore x = 96$.

13. If 6 is substituted for x, the equation is satisfied.

14. If there are x \$2 bills and $(35 - x)$ \$5 bills, then $2x + 5(35 - x) = 115$, $\therefore x = 20$.

15. Substitute $\frac{1}{2}$ for a and the resulting equation gives $x = 15$.

16. If the height is x in., then $45x = 40(x+1)$, $\therefore x = 8$.

17. Multiply by 120, $24(x-4) - 20(x-5) = 5(x-2)$, $\therefore x = 14$.

18. If the smaller is x, then $\frac{1}{23}x + \frac{1}{27}(150 - x) = 6$, $\therefore x = 69$.

19. If they are x and $x+1$, then $(x+1)^2 - x^2 = 51$, $\therefore x = 25$.

20. If the son's age is x years, then the father's is $(x + 30)$ years, $\therefore x + 30 - 5 = 4(x-5)$, $\therefore x = 15$.

21. If $\dfrac{2x+3}{3} + \dfrac{x+5}{7} = 9$, then $x = 9$ and the values are 7, 2.

22. If the nos. are x and $x+1$, it is required to show that $(x+1)^2 - x^2 = x + x + 1$ and that $x^2 + (x+1)^2 = 2x(x+1) + 1$.

23. $x = \frac{1}{3}$. 24. $x = 60$. 25. $x = -\frac{18}{43}$.

26. If x is the greater and $\frac{3}{4} - x$ the less, $3x - 6(\frac{3}{4} - x) = \frac{3}{4}$, $\therefore x = \frac{7}{12}$.

27. Multiply by 15, $3x - 9 + 10 + 5x - 1 + 2x = 0$. $\therefore x = 0$.

28. If x miles is the distance, then $\dfrac{x}{3} + \dfrac{x}{33} = 4$, $\therefore x = 11$.

29. If x is the no., the results in order are $2x$, $2x + 12$, $x + 6$, 6.

30. Multiply by 12, $8x + 3x + 3 - 6x + 6 = 12x - 96$, $\therefore x = 15$.

31. If it is x min. to 10, then 45 min. ago it was $[120-(x+45)]$ min. past 8, $\therefore 120-(x+45)=2x$, $\therefore x=25$.

32. When simplified, the x cancels and $a=24$.

33. $12x^2-10x+2-12x^2+14x-4=4$, $\therefore x=1\frac{1}{2}$.

34. If the dimensions of the first are x and $x+5$, then the second are $x-3$ and $x+10$, $\therefore x(x+5)=(x-3)(x+10)$, $\therefore x=15$.

35. $x^2+3x+2+x^2+7x+12=2x^2+24x$, $\therefore x=1$.

36. If a daughter gets $\$x$, a son gets $\$2x$, and the wife gets $\$(3x+4x+500)$, $\therefore 7x+7x+500=7500$, $\therefore x=500$.

37. Multiply by 18, $6x-12+12x+15-14x+16=0$, $\therefore x=-4\frac{3}{4}$.

38. If the no. is x, then $(x+1)^2-x^2=37$, $\therefore x=18$.

39. If $\frac{2x+1}{3}-\frac{3x-2}{4}=\frac{x-2}{6}$, then $x=4\frac{2}{3}$.

40. If x mi. is the distance, then $\frac{1}{3}x+\frac{1}{10}x=6\frac{1}{15}$, $\therefore x=14$.

41. If he invested $\$x$, then $\frac{1}{3}\times\frac{3}{100}x+\frac{1}{4}\times\frac{4}{100}x+\frac{1}{5}\times\frac{5}{100}x+\frac{13}{60}\times\frac{6}{100}x=516$, $\therefore x=12000$.

42. If the numbers are x and y, then $(x+y)(x-y)=x^2-y^2$.

Exercise 41 — Page 81

1. Subtract (2) from (1) and $y=3$, then $x=2$.

2. Subtract and $y=2$, then $x=1$.

3. Multiply (2) by 2 and subtract and $y=1$, $x=3$.

4. Add and $x=8$, then $y=3$.

5. Mult. (2) by 2 and subtract and $y=1$, then $x=4$.

6. Mult. (1) by 3 and (2) by 2 and subtract and $x=4$, $y=2$.

7. Mult. (1) by 2 and (2) by 3 and subtract and $y=0$, $x=6$.

8. Mult. (2) by 2 and subtract and $x=5$, $y=-1$.

9. Mult. (1) by 3 and (2) by 2 and subtract and $x=10$, $y=3$.

10. Add and $x=3\frac{1}{2}$, $y=\frac{1}{2}$.

11. Mult. (1) by 3 and subtract, then $x=\frac{2}{3}$, $y=\frac{3}{4}$.

12. Mult. (1) by 2 and (2) by 3 and add, then $x=4$, $y=6$.

13. Mult. (1) by 3 and (2) by 4 and add, then $x=8$, $y=2$.

14. Mult. (1) by 3 and (2) by 2 and subtract, then $y=-2$, $x=5$.

15. Mult. (1) by 2 and (2) by 3 and add, then $y=4$, $x=10$.

16. Mult. (1) by 3 and (2) by 2 and subtract, then $x=9$, $y=10$.

17. Rearrange in the form $x-3y=20$, $2x-y=20$, $x=8$, $y=-4$.

18. $3x-2y=0$, $2x-5y=-33$, $x=6$, $y=9$.

19. Mult. (1) by 7 and subtract, then $y = 5$, $x = 105$.

20. $2x + 3y = 17$, $5x - y = 17$. $x = 4$, $y = 3$.

21. $4x - 5y = -10$, $14x - 10y = 10$. $x = 5$, $y = 6$.

22. Solving, $x = 2\frac{1}{2}$, $y = 4\frac{1}{2}$.

23. If $2x - 5y = 31$, $9x - 6y = 57$, then $x = 3$, $y = -5$.

24. $x + 2y = 1$, $7x + 11y = 13$. $x = 5$, $y = -2$.

25. If $3x + 4y = 39$, $5x - 2y = 13$, $x = 5$, $y = 6$.

26. If $16x - y = 6$, $4x + 2y = 6$, $x = \frac{1}{2}$, $y = 2$.

27. $5x + y = 31$, $11x - 13y = 53$, $x = 6$, $y = 1$.

Exercise 42 — Page 82

1. Multiply (1) by 6 and (2) by 2, then subtract and $x = 4$, $y = 3$.

2. Multiply (1) by 6 and (2) by 2, then $x + 6y = 36$, $2x + y = 28$. Eliminate x and $y = 4$, then $x = 12$.

3. $x + y = 18$, $x - y = 12$; $x = 15$, $y = 3$.

4. $5x + 6y = 420$, $2x + 9y = 432$; $x = 36$, $y = 40$.

5. $x + 6y = 4$, $x + 4y = 0$; $x = -8$, $y = 2$.

6. $3x + 8y = 360$, $5x - 4y = 80$; $x = 40$, $y = 30$.

7. $9x - 4y = 12$, $8x + 9y = 312$; $x = 12$, $y = 24$.

8. $8x + 3y = 984$, $3x - 4y = 0$; $x = 96$, $y = 72$.

9. $5x + 81y = 819$, $81x + 5y = 1503$; $x = 18$, $y = 9$.

10. $3x + 2y = 48$, $3x - y = 12$; $x = 8$, $y = 12$.

11. $3x - 2y = 0$, $11x - 9y = -80$; $x = 32$, $y = 48$.

12. $4x + 3y = 72$, $4y - x + y = 28$ or $x - 5y = -28$; $x = 12$, $y = 8$.

13. $30x + 50y = 23$, $60x + 50y = 26$; $x = .1$, $y = .4$.

14. $x + 30y = 26$, $10x - 16y = 102$; $x = 11$, $y = \frac{1}{2}$.

15. $5x + 3y = 2900$, $3x - 4y = 0$; $x = 400$, $y = 300$.

16. $3x = y - 1$ or $3x - y = -1$, $2x = x - 9$ or $x = -9$, $y = -26$.

17. $3x - 5y = 45$, $6x - 7y = 126$; $x = 35$, $y = 12$.

18. $x - y = 12 - 2y$ or $x + y = 12$, $2x = 30 - 5y$ or $2x + 5y = 30$; $x = 10$, $y = 2$.

19. $6x + y = 42$, $x + 6y = 42$; $x = 6$, $y = 6$.

20. $4x - 3y = 72$, $3x + 7y = -20$; $x = 12$, $y = -8$.

21. $3x - 3y + 2x + 2y = 15$ or $5x - y = 15$,
$3x + 3y + 2x - 2y = 25$ or $5x + y = 25$; $x = 4$, $y = 5$.

22. $6x + 3y - 3 = 6y + 2x + 2y$ or $4x - 5y = 3$,
$6y - 3x + 3 = 2x + y + 3$ or $5x - 5y = 0$; $x = -3$, $y = -3$.

23. $3x + y = 3y - 6$ or $3x - 2y = -6$,
$4y + x = 4x + 24$ or $3x - 4y = -24$; $x = 4$, $y = 9$.

24. $5x + 5y - 7x + 7y = 26$ or $12y - 2x = 26$,
$9x + 21y = 24x - 4y$ or $25y - 15x = 0$; $x = 5$, $y = 3$.

25. $8x - 7y = 12$, $11x - 10y = 12$; $x = 12$, $y = 12$.

26. $x + 1 = 5(3y - 5)$ or $x - 15y = -26$,
$4(3y - 5) = x - y$ or $x - 13y = -20$; $x = 19$, $y = 3$.

27. $15y - 8x = -960$, $6y + 7x = -231$; $x = 15$, $y = -56$.

Exercise 43 — Page 83

1. If x is the greater and y the less, then $x + y = 40$, $x - y = 12$, $\therefore x = 26$, $y = 14$.

2. If they are x and y, $x + y = 19$, $3x + 4y = 64$; $x = 12$, $y = 7$.

3. If they cost x^c and y^c, $4x + 7y = 242$, $5x + 3y = 268$; $x = 50$, $y = 6$.

4. If they are x and y, $7x - 2y = 23$, $5x - 3y = 136$; $x = 11$, $y = 27$.

5. If they cost $\$x$ and $\$y$, $5x + 6y = 840$, $3x + 2y = 440$; $x = 120$.

6. If they cost $\$x$ and $\$y$, $9x + 7y = 156$, $10x + 2y = 156$; $x = 15$, $y = 3$.

7. If they earn $\$x$ and $\$y$, $3x + 4y = 41$, $5x + 2y = 45$; $x = 7$, $y = 5$.

8. If they are x and y, $\frac{1}{2}x + \frac{1}{3}y = 26$, $\frac{1}{9}x + \frac{1}{6}y = 8$; $x = 36$, $y = 24$.

9. If the prices are x^c and y^c, $3x - 5y = 20$, $2x + y = 230$; $x = 90$, $y = 50$.

10. If the father's age is x years and the son's y years, then $x + 10 = 2(y + 10)$, $x - 8 = 8(y - 8)$; $x = 32$, $y = 11$.

11. $x + y + 3(x - y) = 18$, $2(x + y) + x - y = 26$; $x = 7$, $y = 5$.

12. If the nos. are x and y, $x + y = 33$, $35x + 25y = 945$; $x = 12$, $y = 21$.

13. $\frac{1}{20}x - \frac{3}{50}y = 3$, $\frac{7}{100}y - \frac{1}{25}x = 7\frac{1}{2}$; $x = 600$, $y = 450$.

14. If an algebra costs x^c and an arithmetic y^c, then $3x + 4y = 295$, $2x + 3y = 210$; $x = 45$, $y = 40$. $\therefore$ 6 algebras and 2 arithmetics cost \$3.50.

15. If there were x of the first and y of the second, $x + y = 10$, $5x + 4y = 46$; $x = 6$, $y = 4$.

16. If there are x double and y single, $x + y = 25$, $2x + y = 42$; $x = 17$.

17. If they cost $\$x$ and $\$y$, $8x + 50y = 900$, $\frac{6}{5} \times 8x + \frac{11}{10} \times 50y = 1030$; $x = 50$.

18. If a man receives $\$x$ and a boy $\$y$ then $10x + 8x = 37$, $4x - 6y = 1$; $x = 2\frac{1}{2}$, $y = 1\frac{1}{2}$.

19. If he paid x^c and y^c, $20x + 15y = 3600$, $15x + 25y = 3250$; $x = 150$, $y = 40$.

20. $x+8=2y,\ y+31=3x$; $x=14,\ y=11$.

21. If there were x and y acres, $x+y=100,\ 37x+45y=4220$; $x=65,\ y=35$.

22. $7x+5y=332,\ 51(x-y)=408$; $x=31,\ y=23$.

23. $\frac{1}{3}(x+y)=20,\ \frac{1}{2}(x-y)=7$; $x=37,\ y=23$.

24. If there were x lb. of tea and y lb. of sugar, $60x+30y=9600$, $75x+35y=11700$, $x=100,\ y=120$.

25. $3x-2y=90,\ 2x+3y=255$; $x=60,\ y=45$.

26. If the numerators are x and y, then $\frac{x}{2}+\frac{y}{5}=2.9,\ \frac{y}{2}+\frac{x}{5}=4.1$; $x=3,\ y=7$.

27. If they are x and y, $x+y=142,\ \frac{1}{17}x+\frac{1}{19}y=8$; $x=85,\ y=57$.

28. If the parts contained x acres and y acres, then $6x+8y=650$, $8x+6y=750$; $x=75,\ y=25$.

29. If their present ages are x years and y years, then $x-3=\frac{1}{2}y$, $x+7+y+7=77$; $x=23,\ y=40$.

30. Suppose he went x miles the first day and $x-y$ the second, then $x+x-y=136,\ x+x-y+x-2y+x-3y=240$; $x=72,\ y=8$.

Exercise 44 — Page 85

1. Multiply (1) by 3 and (2) by 2 and subtract, then $y=8,\ x=7$.

2. If there were x hits, $5x-2(20-x)=51$; $x=13$.

3. Multiply (1) by 3 and (2) by 2 and add, then $x=5,\ y=11$.

4. If x passed, then $1000-x$ failed. The total no. of marks was 53000, $\therefore\ 65x+25(1000-x)=53000$; $x=700$.

5. $-x+10y=0,\ 3x+14y=88$; $x=20,\ y=2$.

6. If A received x, then B received $x-384$, $\therefore\ \frac{3}{11}(2x-384)=384$; $x=896$.

7. $3x-2y=19,\ 3x-4y=5$; $x=11,\ y=7$.

8. If the first part is \$$x$, $\frac{3}{100}x=\frac{4}{100}(5600-x)$; $x=3200$.

9. $4x+3y=0,\ 3x-7y=37$; $x=3,\ y=-4$.

10. If x is the larger, $\frac{1}{6}x=\frac{1}{5}(x-11)+1$; $x=36$.

11. $5x+3y=74,\ 6x+2y=76$; $x=10,\ y=8$.

12. $\frac{3}{100}x+\frac{4}{100}y=93,\ \frac{5}{100}x+\frac{3}{100}y=111$; $x=1500,\ y=1200$.

13. $x=3,\ y=5$. **14.** 64, 36. **15.** $x=2,\ y=-1$. **16.** \$48. **17.** $x=7,\ y=1$. **19.** \$150, \$250. **20.** $x=2,\ y=-2$. **21.** 7, 17. **22.** $x=13,\ y=7$.

23. If B's wages are $\$x$, then A's are $\$\frac{3}{2}x$. If B spends $\$y$, then A spends $\$2y$. $\therefore \frac{3}{2}x - 2y = 5$, $x - y = 10$; $x = 30$.

24. If $y = \frac{3}{2}x$, then $15y = 22\frac{1}{2}x$, $\therefore 23x + 22\frac{1}{2}x = 91$; $x = 2$.

25. If his wife's age was x years, then his was $\frac{6}{5}x$ years. $\therefore \frac{6}{5}x + 8 = \frac{8}{7}(x + 8)$, $\therefore x = 20$.

26. If $15x - 6y = 6x + 12y$, $9x = 18y$ or $x = 2y$.

27. If the no. of acres is x and y, then $5x + 5y = 1100$ and $4x + 6y = 1120$; $x = 100$, $y = 120$.

28. If $4x + 3y = 108$, $8x + 33y = 432$; $x = 21$, $y = 8$; $\frac{1}{7}x + \frac{1}{2}y = 7$.

29. If their ages are x and y, $y - 7 = 3(x - 7)$, $y + 5 = 2(x + 5)$; $x = 19$, $y = 43$.

30. $10x + 8y = 35$, $14(x - y) = 88x - 33$; $x = -\frac{1}{2}$, $y = 5$.

31. $6x + 10 = 20 + y + 1$ or $6x - y = 11$, $4y + 32 = 36 + 3x + 15$ or $3x - 4y = -19$; $x = 3$, $y = 7$.

Exercise 45 — Page 89

1. $2(2x + 3)$. **2.** $3(a - 3)$. **3.** $5(x - 2y)$. **4.** $x(a + 3)$.
5. $b(x - y)$. **6.** $x(x + 1)$. **7.** $p(7p - 6)$. **8.** $3y(2y + 1)$.
9. $2x^2(4x - 1)$. **10.** $2(y + 2)$. **11.** $6(m - 2)$. **12.** $3(x^2 - 5)$.
13. $a(b + c)$. **14.** $m(a - b)$. **15.** $a(b + c + 1)$. **16.** $m(x + y - z)$.
17. $x(x - 7)$. **18.** $5a(a + 2b)$. **19.** $2x(2x^2 + 3x + 1)$.
20. $a^2(x + y - 1)$. **21.** $5x(3x - 2y)$. **22.** $2a(x - 2y + 3z)$.
23. $x(x - 3xy + y^2)$. **24.** $2ab(2 + 3ab - 4c)$. **25.** $(x + y)(a + b)$.
26. $(a - b)(x + y)$. **27.** $2(b - c)(x - 1)$.

Exercise 46 — Page 90

1. $x^2 + 3x + 2$. **2.** $x^2 + 16x + 55$ **3.** $x^2 - 7x + 12$.
4. $x^2 - 17x + 60$. **5.** $y^2 - y - 30$. **6.** $m^2 + 2m - 8$.
7. $a^2 + 3ab + 2b^2$. **8.** $x^2 - 5xy + 6y^2$. **9.** $x^2 + xy - 12y^2$.
10. $y^2 - 25x^2$. **11.** $p^2 + 5pq - 66q^2$. **12.** $a^2 - 2\frac{1}{2}a + 1$.
13. $a^2b^2 - 4ab + 3$. **14.** $x^2y^2 - 49$. **15.** $p^2q^2 - r^2$.
16. $a^2x^2 - 5abxy + 6b^2y^2$. **17.** $a^2 + 3a + 2$. **18.** $x^2 - 5xy + 4y^2$.
19. $p^2 - q^2$. **20.** $x^2 - xy - 6y^2$. **21.** $m^2 - mn - 20n^2$. **22.** $b^2 - 3\frac{1}{3}b + 1$.

23. $3x + 6 + 6x - 2 - x + 3 = 8x + 7$. To check, put $x = 1$, then given expression $= 3 \times 3 + 2 \times 2 + 2 = 15$ and the result is $8 + 7 = 15$.

24. $x^2 + 3x + 2 + x^2 + 5x + 6 = 2x^2 + 8x + 8$.

25. $y^2 + y - 6 + y^2 - y - 20 = 2y^2 - 26$.

26. $x^2 + 2x + 1 + x^2 - 1 + x^2 - x - 2 = 3x^2 + x - 2$.

27. $2m^2 + 6m + 4 + 3m^2 - 9m + 6 = 5m^2 - 3m + 10$.

28. $4x^2 + 16x + 12 - x^2 - 13x - 12 = 3x^2 + 3x$.

Exercise 47 — Page 91

1. $(x+7)(x+1)$. **2.** $(x+5)(x+1)$. **3.** $(y+5)(y+3)$. **4.** $(a+1)(a+21)$. **5.** $(x+6)(x+2)$. **6.** $(b+2)(b+12)$. **7.** $(a+b)(a+2b)$. **8.** $(m+5n)(m+2n)$. **9.** $(y+x)(y+39x)$. **10.** $(x-3)(x-2)$. **11.** $(x-6)(x-1)$. **12.** $(x-1)(x-11)$. **13.** $(x-y)(x-3y)$. **14.** $(a-7b)(a-4b)$. **15.** $(m-4n)(m-3n)$. **16.** $(x-5)(x+4)$. **17.** $(y-6)(y+5)$. **18.** $(a+6)(a-5)$. **19.** $(x-7)(x+2)$. **20.** $(m-10)(m+4)$. **21.** $(x+2)(x-12)$. **22.** $(ab+5)(ab+3)$. **23.** $(xy-5)(xy-6)$. **24.** $(x^2-9)(x^2-1)$. **25.** $(a+3)^2$. **26.** $(x-7)^2$. **27.** $(y-6)^2$. **28.** $(a+1)+(a-1)$. **29.** $(m-2)-(m-3)$. **30.** $(x+2)(x+1)(x-5)\div(x-5)(x+2)$. **31.** $x-2+x-2+x-4$. **33.** $2(x^2-5x+6)=2(x-3)(x-2)$. **34.** $3(a^2+a-12)=3(a+4)(a-3)$.

35. $x(x^2-8x+7)=x(x-7)(x-1)$.

36. The factors might be $(x-6)(x+1)$, then $m=-5$ or $(x+6)(x-1)$, then $m=5$ or $(x-3)(x+2)$, and so on.

37. It is not factored since it is a difference not a product.

Exercise 48 — Page 93

1. ± 6. **2.** ± 9. **3.** ± 11. **4.** $\pm 1\frac{1}{2}$. **5.** $\pm y$. **6.** $\pm bc$. **7.** $\pm 5a$. **8.** $\pm 8xy$. **9.** $\pm \frac{1}{2}a$. **10.** $\pm \frac{1}{3}mn$. **11.** $\pm \frac{2}{3}p^2$. **12.** $\pm 2\frac{1}{2}x$. **13.** ± 3. **14.** ± 5. **15.** $\pm 2a$. **16.** $\pm ab$. **17.** $x+2=\pm 9$, $x=-2\pm 9=7$ or -11. **18.** $x-3=\pm 7$, $x=10$ or -4. **19.** $x-5=0$, $x=5$. **20.** $x^2=100$, $x=10$. **21.** $r^2=154\div 3\frac{1}{7}=49$, $r=7$. **22.** $3\frac{1}{7}r^2=616$, $r^2=196$, $r=14$. **23.** $4.3\frac{1}{7}r^2=154$, $r^2=\frac{49}{4}$, $r=3\frac{1}{2}$.

Exercise 49 — Page 95

1. a^2+2a+1. **2.** y^2+4y+4. **3.** m^2-2m+1. **4.** $x^2-8x+16$. **5.** $4a^2+4a+1$. **6.** $1-6x+9x^2$. **7.** $p^2-2pq+q^2$. **8.** $4x^2+12x+9$. **9.** $4a^2-12a+9$. **10.** $m^2-4mn+4n^2$. **11.** $9x^2-12xy+4y^2$. **12.** $16x^2-24ax+9a^2$. **13.** $\frac{1}{4}-x+x^2$. **14.** $4y^2-2y+\frac{1}{4}$. **15.** $9x^2-4x+\frac{4}{9}$. **16.** x^2+4x+4.

17. $x^2+2x+1+x^2-2x+1=2x^2+2$.

18. $a^2-2ab+b^2+a^2+2ab+b^2=2a^2+2b^2$.

19. $4x^2+4x+1+x^2-4x+4=5x^2+5$.

20. $(a^2+2ab+b^2)-(a^2-2ab+b^2)=a^2+2ab+b^2-a^2+2ab-b^2$.

21. $9m^2-6mn+n^2-4m^2-4mn-n^2=5m^2-10mn$.

22. $9x^2+12xy+4y^2-4x^2+12xy-9y^2=5x^2+24xy-5y^2$.

23. $x^2+2x+1+x^2+4x+4+x^2+6x+9=3x^2+12x+14$.

24. $x^2 - 2x + 1 + x^2 - 4x + 4 - x^2 + 6x - 9 = x^2 - 4.$

25. $2(a^2 + 2a + 1) + 3(a^2 - 2a + 1) - 5(a^2 - 4a + 4) = 2a^2 + 4a + 2 + 3a^2 - 6a + 3 - 5a^2 + 20a - 20 = 18a - 15.$

26. $(x - y)^2 + (x + y)^2 + (x - 2y)^2 = x^2 - 2xy + y^2 + x^2 + 2xy + y^2 + x^2 - 4xy + 4y^2 = 3x^2 - 4xy + 6y^2.$

27. $x^2+2x+1+x^2-4x+4+x^2-6x+9-3x^2+24x-48 = 16x-34.$

28. $(x+2)^2+(x+3)^2+(x+4)^2=3x^2+18x+29.$ $(x-2)^2+(x-3)^2+(x-4)^2=3x^2-18x+29.$ The difference $= 36x.$

29. $4a^2 - 12ab + 9b^2 + 9a^2 + 12ab + 4b^2 - 4a^2 - 8ab - 4b^2 = 9a^2 - 8ab + 9b^2.$

30. If they are a and $a + 2$, then $(a + 2)^2 - a^2 = 4a + 4 = 2(2a+2).$

31. $\left(x + \frac{2}{x}\right)^2 = x^2 + 4 + \frac{4}{x^2}.$ $\left(x - \frac{2}{x}\right)^2 = x^2 - 4 + \frac{4}{x^2}.$ The diff. $= 8.$

32. If they are $a + 1$, a, $a - 1$, then $(a - 1)^2 + a^2 + (a+1)^2 = 3a^2+2.$

33. $1235^2 = (1234 + 1)^2 = 1234^2 + 2468 + 1 = 1{,}525{,}225.$

34. $(n + \frac{1}{2})^2 = n^2 + n + \frac{1}{4} = n(n + 1) + \frac{1}{4}.$ $(15\frac{1}{2})^2 = 15 \times 16 + \frac{1}{4}.$

Exercise 50 — Page 96

1. $(x + y)^2.$ **2.** $(y - 1)^2.$ **3.** $(2x + 1)^2.$ **4.** $(2a + 5)^2.$ **5.** $(3a - 4)^2.$ **6.** $(4x - 1)^2.$ **7.** $(ab - 1)^2.$ **8.** $(1 - 3y)^2.$ **9.** $(3x - 3y)^2.$ **10.** $(abc - 1)^2.$ **11.** $(x + \frac{1}{2})^2.$ **12.** $(y - \frac{1}{2}x)^2.$ **13.** $3a + 2.$ **14.** $x - 2y.$ **15.** $1 - 3x.$ **16.** $2ab - 5.$ **17.** $2m + \frac{1}{2}.$ **18.** $a - 7b.$ **19.** $2 - a.$ **20.** $3 - 2x.$ **21.** $3x-5y.$ **22.** $2ab.$ **23.** $4xy.$ **24.** $+9.$ **25.** $12m.$ **26.** $+9.$ **27.** $x^2.$

28. It must be the square of $4a - 2$ or $4a + 2$. $m = \pm 16.$

29. $3x + 1 = 31$ when $x = 10$ and $9x^2 + 6x + 1 = 961$ and $31^2 = 961.$

30. $x + 1 + x + 5 = 14,$ $x = 4$; $3(x - 2) - 2(x + 3) = -2,$ $x = 10$; $3x + 1 + 2x + 1 + x - 1 = 13,$ $x = 2.$

31. First side $= 2(a - 3) - (a - 2) = a - 4.$ Second $= 3(a - 1) - (2a + 1) = a - 4.$

Exercise 51 — Page 98

1. $m^2 - n^2.$ **2.** $p^2 - q^2.$ **3.** $a^2 - 4.$ **4.** $x^2 - 25.$ **5.** $4a^2 - 1.$ **6.** $9x^2 - 4.$ **7.** $4a^2 - 9x^2.$ **8.** $16x^2 - 25y^2.$ **9.** $x^2 - \frac{1}{4}.$ **10.** $x^4 - 4y^2.$ **11.** $25x^2 - a^2b^2.$ **12.** $4x^2 - \frac{1}{9}y^2.$ **13.** $(x+1)(x-1).$ **14.** $(y + 2)(y-2).$ **15.** $(a + 2b)(a - 2b).$ **16.** $(2m+n)(2m-n).$ **17.** $(2p + 3q)(2p - 3q).$ **18.** $(x + \frac{1}{2})(x - \frac{1}{2}).$ **19.** $(3+x)(3-x).$ **20.** $(1+4ab)(1-4ab).$ **21.** $(5+7x)(5-7x).$ **22.** $(a^2+5)(a^2-5).$ **23.** $(ab + 7)(ab - 7).$ **24.** $(99 + 98)(99 - 98).$ **25.** $a^2 - 4 + 4a^2 - 1.$ **26.** $4a^2 - 9b^2 - a^2 + b^2.$ **27.** $2x^2 - 18y^2 + 18y^2 - 2x^2.$ **28.** $2p^2 - 4pq + 2q^2 + 3p^2 - 3q^2 - 5p^2 + 20q^2.$

30. First product $= x^4 - 1$, second $= x^4 - 16$. Diff. $= 15$.
31. $3(x^2 - y^2) = 3(x + y)(x - y)$. **32.** $5(x^2 - 4) = 5(x - 2)(x + 2)$. **33.** $a(a^2 - 1) = a(a - 1)(a + 1)$. **34.** $m(x^2 - a^2) = m(x - a)(x + a)$. **35.** $5(1 - 9p^2) = 5(1 - 3p)(1 + 3p)$. **36.** $(x^2 - y^2)(x^2 + y^2) = (x - y)(x + y)(x^2 + y^2)$. **37.** $\pi(R^2 - r^2) = \pi(R + r)(R - r)$. **38.** $(x^2 - 1)(a + b) = (x - 1)(x + 1)(a + b)$. **39.** Because one factor of the difference of the squares is the difference of the numbers, which is unity.

40. $(a - b)(a + b)(a - 2b)(a - 3b) \div (a - b)(a - 2b) = (a + b)(a - 3b)$.

41. $x + y + x - y = 2x$; $x + 4 - (x - 3) = 7$.

42. $x - 1 + x + 3 = 10$, $x = 4$; $2x^2 - 50 = 15 + x^2 - 1$, $x^2 = 64$.

Exercise 52 — Page 100

1. $98^2 = (100 - 2)^2 = 10000 - 400 + 4$; $58^2 = (60 - 2)^2 = 3600 - 240 + 4$.

2. $91 \times 89 = (90 + 1)(90 - 1) = 8100 - 1$; $47 \times 53 = 50^2 - 3^2 = 2500 - 9$.

3. $52^2 - 48^2 = (52 + 48)(52 - 48)$; $673^2 - 573^2 = 1246 \times 100 = 124600$.

4. $x^2 = (13 + 12)(13 - 12) = 25$ or $x = \pm 5$; $x = \pm 7$.

5. $7x^2 = (64 + 57)(64 - 57) = 121 \times 7$ or $x^2 = 121$, $x = \pm 11$.

6. $a^2 - b^2 = 81, 25, 400, 6600, 29\,m^2, 3.5$.

7. $\pi(R^2 - r^2) = 22, 462, 154, 2090, 1056\,a^2, 15.84$.

Exercise 53 — Page 101

1. $3(x + 2y)$. **2.** $4(m - 3n)$. **3.** $x(a - b)$. **4.** $3c(2a - b)$. **5.** $(x + 2)^2$. **6.** $(a - 1)^2$. **7.** $(2a - 3x)(2a + 3x)$. **8.** $(x - 1)(x - 2)$. **9.** $(y + 10)(y - 11)$. **10.** $2(a + 3)(a - 3)$. **11.** $(10p - 9q)(10p + 9q)$. **12.** $(a - 20)(a + 1)$. **13.** $2(a + 1)(a + 2)$. **14.** $3(10 + x)(10 - x)$. **15.** $(x + y + 1)(x + y - 1)$. **16.** $(a - b)(a + b)(a^2 + b^2)$. **17.** ± 10, $\pm \frac{1}{2}$, $\pm \frac{2}{3}$, $\pm \frac{1}{2}$. **18.** $4x^2 - 12x + 9$, $25x^2 - 60x + 36$, $16x^2 - 24xy + 9y^2$, $a^2 - \frac{1}{2}a + \frac{1}{16}$, $b^2c^2 - bc + \frac{1}{4}$. **19.** $a + 3$, $p - 4$, $mn - 5$, $a - \frac{1}{2}$, $4x - 5y$. **20.** $9a$. **21.** $\pm 30xy$. **26.** $2(x^4 - 16) = 2(x^2 - 4)(x^2 + 4) = 2(x - 2)(x + 2)(x^2 + 4)$; $(a^2 - 4)(a^2 - 9) = (a - 2)(a + 2)(a - 3)(a + 3)$; $2m(m^2 - 9) = 2m(m + 3)(m - 3)$; $(x^2 - y^2)(a^2 - b^2) = (x - y)(x + y)(a + b)(a - b)$.

29. $ab = 92 \times 88 = (90 + 2)(90 - 2) = 8100 - 4 = 8096$.
$a^2 - b^2 = 92^2 - 88^2 = (92 + 88)(92 - 88) = 180 \times 4 = 720$.
$a^2 + b^2 = (90 + 2)^2 + (90 - 2)^2 = 90^2 + 360 + 4 + 90^2 - 360 + 4 = 16208$.

31. If the factors are $(x+42)(x+1)$, then $b=43$; if $(x+14)(x+3)$, then $b=17$; if $(x+7)(x+6)$, then $b=13$; if $(x+21)(x+2)$, then $b=23$.

33. $427^2=(426+1)^2=426^2+852+1=181476+853=182329$.

Exercise 54 — Page 104

1. 3. **2.** 8. **3.** 2. **4.** $3x$. **5.** $2x$. **6.** ab. **7.** x^2. **8.** $5a$.

9. $17x^2$. **10.** a. **11.** $2x$. **12.** $a+b$.

13. $2(a+2b)$, $3(a+2b)$. The H. C. F. is now evident.

14. $(a+b)(a-b)$, $b(a-b)$. **15.** $(m-n)(m+n)$, $(m-n)^2$.

16. $x(x+y)$, $y(x+y)$, $(x+y)^2$. **17.** $n(m+2)$, $(m+1)(m+2)$.

18. $(a-1)(a-2)$, $(a-2)(a-3)$.

19. $(x-3)(x+3)$, $(x-3)(x-4)$, $(x-3)(x-1)$.

20. $(y+3)(y-1)$, $(y+2)(y-1)$. **21.** $(a+b)^2$, $2(a-b)(a+b)$.

22. $(x-5)^2$, $3(x-5)(x+5)$. **23.** $2b(3a+2b)$, $2a(3a+2b)$.

24. $a(a-1)^2$, $a(a-1)(a+2)$.

25. The other factor of $a^2+mab+b^2$ must be $a+b$, and, therefore, $m=2$; the other factor of $a^2+nab+2b^2$ is $a+2b$ and $n=3$.

26. $a^2b=16$, $ab^2=32$, and the G. C. M. $=16$, while $ab=8$.

Exercise 55 — Page 106

6. $\frac{2}{3}$. **7.** $\frac{x}{2}$. **8.** $\frac{3a}{5}$. **9.** $\frac{b}{2c}$. **10.** $\frac{b}{c}$. **11.** $\frac{2x}{3}$. **12.** $\frac{2m}{3}$.

13. $7a$. **14.** $\frac{a+2}{3}$. **15.** $\frac{a+2}{4}$. **16.** $\frac{2}{x-4}$. **17.** $\frac{a}{b}$.

18. $\frac{x}{x-1}$. **19.** $\frac{x-1}{x-3}$. **20.** $\frac{x-y}{2a}$. **21.** $\frac{a}{a-b}$.

22. $\frac{(x+1)(x-1)}{x(x-1)}=\frac{x+1}{x}$. **23.** $\frac{y(y-1)}{(y-1)^2}=\frac{y}{y-1}$. **24.** $\frac{(x-1)(x-2)}{(x-1)(x-3)}$ $=\frac{x-2}{x-3}$. **25.** $\frac{(m+3)(m+4)}{m(m+4)}=\frac{m+3}{m}$. **26.** $\frac{(a-b)(a+b)}{(a+b)(a+4b)}$ $=\frac{a-b}{a+4b}$. **27.** $\frac{3(x-y)(x+y)}{3(x+y)(x+2y)}=\frac{x-y}{x+2y}$. **28.** $\frac{a(a-1)(a+1)}{a(a-1)}$ $=a+1$. **29.** $\frac{2(a-1)(a+1)(a^2+1)}{4(a-1)(a+1)}=\frac{a^2+1}{2}$.

30. $\frac{x(x-1)(x+1)(x^2+1)}{x(x-1)(x+1)}=x^2+1$.

Exercise 56 — Page 107

1. $\frac{1}{2}$. **2.** $\frac{4}{9}$. **3.** $\frac{3a}{2c}$. **4.** 1. **5.** $\frac{az}{cx}$. **6.** $\frac{1}{2}$. **7.** $\frac{ad}{bc}$.

8. $\frac{5}{6y}$. **9.** $\frac{a}{10b}$. **12.** $\frac{x(x+y)}{a(a+b)} \times \frac{b(a+b)}{y(x+y)} = \frac{bx}{ay}$.

13. $\frac{(x-1)(x+1)}{(x-2)(x+2)} \times \frac{(x-2)(x-3)}{(x-3)(x-1)} = \frac{x+1}{x+2}$.

14. $\frac{(a-1)(a-2)}{(a-2)(a-3)} \times \frac{(a-3)(a-4)}{(a-1)(a-5)} \times \frac{(a-3)(a-1)}{(a-1)(a-4)} = \frac{a-3}{a-5}$.

15. $\frac{(a-b)(a+b)}{(a-b)(a-2b)} \times \frac{(a-2b)(a+4b)}{(a-3b)(a+b)} \times \frac{(a-b)(a-3b)}{(a+4b)(a-b)} = 1$.

Exercise 57 — Page 108

1. 60. **2.** 60. **3.** $12a$. **4.** a^2b. **5.** x^2y^2. **6.** $6abc$. **7.** $30a^2$.
8. $12a^3$. **9.** $12a^2b^2$. **10.** a^2, $a(a+1)$. **11.** $3x$, $3x(x+2)$.
12. $a(b+c)$, $b(b+c)$. **13.** $2(x+1)$, $(x+1)(x-1)$.
14. $x(x+y)$, $(x+y)^2$. **15.** $(x+1)(x-1)$, $(x-1)(x-2)$.
16. $a(a-b)$, $b(a-b)$. **17.** $(a-b)(a+b)$, $(a-b)^2$.
18. $x(x-1)$, $x(x-1)(x+1)$. **19.** $2x$, $4(x+1)$, $2(x+1)(x-1)$.
20. $(y-1)(y-2)$, $(y-2)(y+1)$, $(y-1)(y+1)$.

21. The product $=(x-1)(x+2)^2(x-3)$; H. C. F. $= x+2$; L. C. M. $=(x-1)(x+2)(x-3)$.

Exercise 58 — Page 109

1. $\frac{6}{9}, \frac{5}{9}$. **2.** $\frac{3b}{4b}, \frac{a}{4b}$. **3.** $\frac{a}{a^2}, \frac{1}{a^2}$. **4.** $\frac{4}{6x}, \frac{9}{6x}$. **5.** $\frac{9a}{12}, \frac{16a}{12}$.

6. $\frac{m^2}{mn}, \frac{n^2}{mn}$. **7.** $\frac{c}{ac}, \frac{ab}{ac}$. **8.** $\frac{2a}{a^2}, \frac{3b}{a^2}$. **9.** $\frac{8a^2}{12a^3}, \frac{15a}{12a^3}, \frac{18}{12a^3}$.

10. $\frac{a^2c}{abc}, \frac{b^2a}{abc}, \frac{c^2b}{abc}$. **11.** $\frac{12cx^2}{6bc}, \frac{9b^2cx}{6bc}, \frac{8}{6bc}$. **12.** $\frac{2x^2}{6xy^2}, \frac{8y}{6xy^2}, \frac{3by^2}{6xy^2}$.

13. $\frac{4cx}{12abc}, \frac{6ay}{12abc}, \frac{3bz}{12abc}$. **14.** $\frac{6a+6}{6a}, \frac{3a-3}{6a}, \frac{2a+4}{6a}$.

15. $\frac{4a+3a}{12} = \frac{7a}{12}$. **16.** $\frac{3(a+b)+2(a-b)}{6} = \frac{5a+b}{6}$.

17. $\frac{5(a+4)+3(5-a)}{15} = \frac{2a+35}{15}$. **18.** $\frac{5(a-x)-3a}{15} = \frac{2a-5x}{15}$

19. $\frac{3(x+1)+2(x-1)}{x^2-1} = \frac{5x+1}{x^2-1}$.

20. $\frac{6(a+b)+4(b+c)+3(a+c)}{12} = \frac{9a+10b+7c}{12}$.

21. $\dfrac{1+x+1-x}{1-x^2}=\dfrac{2}{1-x^2}$. **22.** $\dfrac{2(x+3)-(4-x)}{6}$.

23. $\dfrac{2(x+y)+4(x-y)-(x+y)}{8}$.

24. $\dfrac{6(x-y)-4(x-y)+3(x+y)}{12}$. **25.** $\dfrac{x(x+y)-y(x-y)}{x^2-y^2}$.

26. $\dfrac{4(a-4)+4(a+4)-8\,a}{a^2-16}=0$. **27.** $\dfrac{x(a+x)+a(a-x)+a^2-x^2}{ax}$ $=\dfrac{2\,a^2}{ax}=\dfrac{2\,a}{x}$.

28. $\dfrac{x}{3(x+2)}-\dfrac{x}{2(x+2)}=\dfrac{2\,x-3\,x}{6(x+2)}=\dfrac{-x}{6(x+2)}$.

29. $\dfrac{2}{a(a-b)}-\dfrac{3}{b(a-b)}=\dfrac{2\,b-3\,a}{ab(a-b)}$. **30.** $\dfrac{1}{(a+1)(a+2)}+\dfrac{2}{(a+2)(a+3)}+\dfrac{3}{(a+1)(a+3)}=\dfrac{a+3+2(a+1)+3(a+2)}{(a+1)(a+2)(a+3)}$.

31. $\dfrac{2}{(a+1)(a-1)}+\dfrac{1}{(a+1)(a+2)}-\dfrac{1}{(a-1)(a+2)}=\dfrac{2\,a+2}{(a+1)(a-1)(a+2)}=\dfrac{2}{(a-1)(a+2)}$.

Exercise 59 — Page 111

1. $\dfrac{17}{6}$. **2.** $\dfrac{2+a}{2}$. **3.** $\dfrac{3\,y+x}{y}$. **4.** $\dfrac{4\,a+m}{4}$. **5.** $\dfrac{3\,x-y}{3}$.

6. $\dfrac{ac-b}{c}$. **7.** $\dfrac{nx-m^2}{n}$. **8.** $\dfrac{2\,x^2+3\,y}{x}$. **9.** $\dfrac{abc-bd}{c}$.

10. $\dfrac{ac}{b+c}$. **11.** $\dfrac{x^2}{x-y}$. **12.** $\dfrac{2\,a^2}{a-b}$. **13.** $\dfrac{3\,x^2+3\,x+2}{3\,x}$.

14. $\dfrac{a-3\,b}{2}$. **15.** $\dfrac{2\,x^2}{x+y}$. **16.** $\dfrac{3\,a}{2}+\dfrac{b}{2}$. **17.** $\dfrac{x}{b}+\dfrac{x}{a}$. **18.** $\dfrac{x}{2\,a}-\dfrac{4\,y}{5\,a}$.

19. $\dfrac{2\,a}{b}-\dfrac{b}{a}$. **20.** $\dfrac{1}{21\,b}+\dfrac{1}{3\,a}-\dfrac{c}{7\,ab}$. **21.** $\dfrac{3}{2\,x}-\dfrac{7}{3\,y}$. **22.** $\dfrac{3\,m}{2}-2$.

23. $2\,a-3+\dfrac{c}{3\,b}$. **24.** $a-b+\dfrac{x}{a-b}$.

Exercise 61 — Page 115

10. $a-b+c-d$. **11.** $a+b+c-d$. **12.** $b+d-a-c$.
13. $2\,a$. **14.** $8\,x+3\,y$. **15.** $a-5\,b+5\,c$. **16.** $4\,b-3\,a$.
17. $6\,a+10\,b-10\,c$.

25. $4\,x-12-7\,x+28=6-x$ or $2\,x=10$, $x=5$.

26. $5\,x-8\,x+48-18\,x-12+15\,x=6$ or $6\,x=30$, $x=5$.

27. $6\,x-21-x+14-10\,x-34=30-48\,x+21\,x+149$, $x=10$.

28. $10(27-2\,x)=135-3(7\,x-54)$, $x=27$.

31. $44\,x-15-27+10-15\,x=110$ or $29\,x=142$, $x=4\tfrac{26}{29}$.

Exercise 62 — Page 117

1. $(3a+4c)-2b$, $(3a-2b)+4c$, $3a-(2b-4c)$.

2. $p-q-(r-s)$, $p-(q+r)+s$, $p-r-(q-s)$, $(p+s)-q-r$.

4. $x(a-c)-y(b+d)$. **5.** $x(m-p-a)-y(n-q-b)$.

6. $x(a-3b+5c)-y(a-2b-2c)$. **7.** $x(a-b-d)+y(b-c-d)$.

8. $x(2a-10+6b)-y(3b+5+4a)$.

9. $y(a-b+1)-x(5-b-2a)$.

10. $(a-b)-(c+d)-(e-f)$, $(a-b-c)-(d+e-f)$.

11. $x^2(a+5b)-x(3a+3b-4c)+4a-c$.

12. $x^2(a-2p-7d)-x(b-3q+3c)+c-r-f$.

Exercise 63 — Page 119

1. x^3-5x^2+8x-4. **2.** $6x^3-19x^2+x+6$. **3.** x^3+1.

4. a^3-b^3. **5.** x^4+x^2+1. **6.** $a^5-4a^4-6a^3+3a^2-2a+2$.

7. $3x^4+x^3-16x^2+x+15$. **8.** $4a^2-25a^2b^2+30ab^3-9b^4$.

9. $a^2-b^2-c^2+2bc$.

10.

$$\begin{array}{lr}
x^3+2x^2+4x+8 & \text{Check } (x=1) \\
\underline{x^2-4x+4} & 15 \\
x^5+2x^4+4x^3+8x^2 & \underline{1} \\
\quad -4x^4-8x^3-16x^2-32x & 15 \\
\qquad +4x^3+8x^2+16x+32 & \\
\hline
x^5-2x^4 \qquad -16x+32 &
\end{array}$$

11. $(b^2-b+1)(b^2+b+1)=b^4+b^2+1$. Product $=b^8+b^4+1$.

12.

$$\begin{array}{lr}
x^2-xy+y^2+x+y+1 & \text{Check } (x=y=1) \\
\underline{x+y-1} & 4 \\
x^3-x^2y+xy^2+x^2+xy+x & \underline{1} \\
\quad x^2y-xy^2+xy+y^3+y^2+y & 4 \\
\qquad -x^2+xy-x-y^2-y-1 & \\
\hline
x^3+3xy+y^3-1 &
\end{array}$$

13.

$$\begin{array}{lr}
3-4+7-3 & 3 \\
\underline{1-2-1} & \underline{-2} \\
3-4+7-3 & -6 \\
\quad -6+8-14+6 & \\
\qquad -3+4-7+3 & \\
\hline
3-10+12-13-1+3 & \text{Product} = 3x^5-10x^4 \text{ etc.}
\end{array}$$

14.

$$\begin{array}{lr}
5-6-2-1+2 & -2 \\
\underline{2-3+2} & \underline{1} \\
10-12-4-2+4 & -2 \\
\quad -15+18+6+3-6 & \\
\qquad +10-12-4-2+4 & \\
\hline
10-27+24-8+3-8+4 & \text{Product} = 10x^6-27x^5 \text{ etc.}
\end{array}$$

15. $16x^5 - 12x^4 - 24x^3 + 7x^2 + 11x + 2.$

16. $6x^5 - 13x^4 - 6x^3 + 17x^2 - 4.$

26. The partial products containing $x^2 = -12 - 30 + 55 = 13.$

27. $2x^2 + 10x^2 - 12x^2 = 0.$ **28.** $30x^2 + 84x^2 - 114x^2 = 0.$

29.
$$\begin{array}{r}
1 + x + x^2 + x^3 \\
\underline{1 + 2x + 3x^2 + 4x^3} \\
1 + x + x^2 + x^3 \\
+ 2x + 2x^2 + 2x^3 \\
+ 3x^2 + 3x^3 \\
\underline{+ 4x^3} \\
1 + 3x + 6x^2 + 10x^3
\end{array}$$

33.
$$\begin{array}{r}
2 + 3x + 4x^2 + 5x^3 \\
\underline{1 - 2x + 3x^2 - 4x^3} \\
2 + 3x + 4x^2 + 5x^3 \\
- 4x - 6x^2 - 8x^3 \\
+ 6x^2 + 9x^3 \\
\underline{- 8x^3} \\
2 - x + 4x^2 - 2x^3
\end{array}$$

34. The terms $= 17x^4 + 52x^4 + 63x^4 + 50x^4 + 13x^4 = 195x^4.$

35. $x^4 - 22x^3 + 164x^2 - 488x + 480 = x^4 - 22x^3 + 164x^2 - 458x + 315.$

39. The first product $= x^3 - x^2(p-1) - x(2p^2 + 2p + 1) + 2p.$
The second product $= x^3 + x^2 - x(p^2 - p - 2) + 2p.$
The difference $= px^2 + x(p^2 + 3p + 3).$

40. In (i) all terms must be of the fifth degree and positive, so that a^3b and the sign of ab^4 are wrong. In (ii) $36xy$ and the sign of $54xy^2$ are wrong. In (iii) $-4x$ should be of the second degree and $2y$ and y^2 must have like signs.

41. Put $a = 2145$ and $b = 3525$ in the formula, then $2146 \times 3526 = 2145 \times 3525 + 5670 + 1 = 7{,}566{,}796.$

Exercise 64 — Page 122

1. $x + 2.$ **2.** $a - 2.$ **3.** $a + b.$ **4.** $x - 2.$ **5.** $a + b.$ **6.** $x - 1.$

7.
$$\begin{array}{r|l}
2x - 3 & 6x^2 + x - 15 \,|\, 3x + 5 \\
& \underline{6x^2 - 9x} \\
& \quad 10x - 15 \\
& \quad \underline{10x - 15}
\end{array}$$

8.
$$\begin{array}{r|l}
3x - 4y & 6x^2 + xy - 12y^2 \,|\, 2x + 3y \\
& \underline{6x^2 - 8xy} \\
& \quad 9xy - 12y^2 \\
& \quad \underline{9xy - 12y^2}
\end{array}$$

9.
$$\begin{array}{r|l}
x - 6y & 5x^2 - 31xy + 6y^2 \,|\, 5x - y \\
& \underline{5x^2 - 30xy} \\
& \quad - xy + 6y^2 \\
& \quad \underline{- xy + 6y^2}
\end{array}$$

10.
$$\begin{array}{r|l}
3a + 7b & 9a^2 + 6ab - 35b^2 \,|\, 3a - 5b \\
& \underline{9a^2 + 21ab} \\
& \quad - 15ab - 35b^2 \\
& \quad \underline{- 15ab - 35b^2}
\end{array}$$

11. $x^2 + 14x$. **12.** $4x^2 - x$. **13.** $1 + 3x$. **14.** $3a - 5$.
15. $x^2 + 7x + 12$. **16.** $2a^2 - 3a + 20$. **17.** $x^2 + 2xy + y^2$.
18. $-x^2 + 2xy - y^2$. **19.** $2m^2 - 5m + 3$. **20.** $3x^2 - 4xy - 5y^2$.

21.
$$\begin{array}{l|l|l} \underline{a^2 - a + 3} & a^4 + a^3 + 4a^2 + 3a + 9 & \underline{a^2 + 2a + 3} \\ & \underline{a^4 - a^3 + 3a^2} & \\ & \quad 2a^3 + a^2 + 3a & \\ & \quad \underline{2a^3 - 2a^2 + 6a} & \\ & \qquad 3a^2 - 3a + 9 & \\ & \qquad \underline{3a^2 - 3a + 9} & \end{array}$$

22.
$$\begin{array}{l|l|l} \underline{x^2 + 2x - 3} & x^4 - x^3 - 6x^2 + 15x - 9 & \underline{x^2 - 3x + 3} \\ & \underline{x^4 + 2x^3 - 3x^2} & \\ & \quad -3x^3 - 3x^2 + 15x & \\ & \quad \underline{-3x^3 - 6x^2 + 9x} & \\ & \qquad 3x^2 + 6x - 9 & \\ & \qquad \underline{3x^2 + 6x - 9} & \end{array}$$

23. $x^2 - 3x + 5$. **24.** $x^2 + 5x + 6$. **25.** $x^2 - x - 6$.

26.
$$\begin{array}{l|l|l} \underline{1 - 2 + 1} & 1 - 3 + 3 - 1 & \underline{1 - 1} \\ & \underline{1 - 2 + 1} & \\ & \quad -1 + 2 - 1 & \\ & \quad \underline{-1 + 2 - 1} & \end{array}$$

27.
$$\begin{array}{l|l|l} \underline{2 - 3 - 1} & 6 - 1 - 11 - 10 - 2 & \underline{3 + 4 + 2} \\ & \underline{6 - 9 - 3} & \\ & \quad 8 - 8 - 10 & \\ & \quad \underline{8 - 12 - 4} & \\ & \qquad 4 - 6 - 2 & \\ & \qquad \underline{4 - 6 - 2} & \end{array}$$

28. $a^3 - 3a^2 + 3a + 1$. **29.** $x^2 - x + 3$.

33. $x^2 - xy + y^2 - (x^2 + xy + y^2) = -2xy$. **34.** $a - 1 + a + 1 = 2a$.

35. $3x + 2 - x - 3 = 11$, $x = 6$. **36.** $x = \dfrac{3a^3 - 7a^2 + 3a - 2}{3a^2 - a + 1} = a - 2$.

37. Divisor = dividend ÷ quotient = $a^2 + 3a - 2$.

38. First quotient = $x^4 + x^2y^2 + y^4$, second = $x^2 + xy + y^2$.

39. $ax - b + ax + b = 2ax$. **40.** $x + c$. **41.** $x + p - 1$.

42.
$$\begin{array}{l|l|l} \underline{ax - (b - c)} & a^2x^2 - 2abx + b^2 - c^2 & \underline{ax - (b + c)} \\ & \underline{a^2x^2 - (ab - ac)x} & \\ & \quad -(ab + ac)x + b^2 - c^2 & \\ & \quad \underline{-(ab + ac)x + b^2 - c^2} & \end{array}$$

43.
$$\begin{array}{l|l|l} \underline{ay^2 + ay + 1} & a^2y^3 + (2a^2 + a)y^2 + (a^2 + 2a)y + a + 1 & \underline{ay + (a + 1)} \\ & \underline{a^2y^3 + (a^2)y^2 + (a)y} & \\ & \quad (a^2 + a)y^2 + (a^2 + a)y + a + 1 & \\ & \quad \underline{(a^2 + a)y^2 + (a^2 + a)y + a + 1} & \end{array}$$

Exercise 65 — Page 125

7. The quot. is 1 and the rem. is 1, then $\dfrac{x+2}{x+1}=1+\dfrac{1}{x+1}$.

8. The quot. is 1 and the rem. is $3\,b$, then $\dfrac{a+2\,b}{a-b}=1+\dfrac{3\,b}{a-b}$.

9. The quot. is 2 and the rem. is $-5\,b$, then $\dfrac{2\,a-3\,b}{a+b}=2-\dfrac{5\,b}{a+b}$.

10. The quot. is $5\,x-3$ and the rem. is 3, then $\dfrac{5\,x^2+7\,x-3}{x+2}=5\,x-3$ $+\dfrac{3}{x+2}$.

11.

$$\begin{array}{l|l|l}
\underline{1-x} & 1 & \underline{1+x+x^2+x^3} \\
 & \underline{1-x} & \\
 & \quad x & \\
 & \quad \underline{x-x^2} & \\
 & \qquad x^2 & \\
 & \qquad \underline{x^2-x^3} & \\
 & \qquad\quad x^3 &
\end{array}$$

14.

$$\begin{array}{l|l|l}
\underline{1-a+a^2} & 1+a+2\,a^2 & \underline{1+2\,a+3\,a^2+a^3} \\
 & \underline{1-a+\ \ a^2} & \\
 & \quad 2\,a+\ \ a^2 & \\
 & \quad \underline{2\,a-2\,a^2+2\,a^3} & \\
 & \qquad 3\,a^2-2\,a^3 & \\
 & \qquad \underline{3\,a^2-3\,a^3+3\,a^4} & \\
 & \qquad\qquad a^3-3\,a^4 &
\end{array}$$

15. $a^2-3\,a+7=a\times$ divisor $+\,7$, then divisor $=\dfrac{a^2-3\,a}{a}=a-3$.

16. The rem. is $a-6$, therefore it is exact when $a=6$.

17. The other factor of $x^2-mx+12$ must be $x-4$ to give the $+\,12$.

Exercise 66 — Page 125

1. $3\,x^3+a^2x+3\,a^3$. **2.** $2\,a-\frac{1}{12}\,b$. **3.** $3\,a-5\,b+10\,c-3\,d$. **4.** $3\,a-4\,b+c$. **5.** $\frac{1}{6}\,a+b-c$. **6.** $\frac{1}{2}\,x-\frac{4}{5}\,y+\frac{2}{3}\,z$. **7.** $-2\,a-4\,b$. **8.** $5\,c-4\,b$. **9.** $3\,x^3-3\,x^2+3\,x-3$. **10.** 0. **11.** $-4\,c$. **12.** $1-10\,x+17\,x^2+48\,x^3-30\,x^4$. **13.** $x^3-7\,x-6$. **14.** $2\,x-3$.

15.

$$\begin{array}{l}
a^2+b^2+c^2-ab-bc-ca \\
\underline{a\ +b\ +c\qquad\qquad\qquad\qquad\qquad} \\
a^3+ab^2+ac^2-a^2b-\ \ abc-a^2c \\
\quad -ab^2\qquad\ +a^2b-\ \ abc\qquad +b^3+bc^2-b^2c \\
\underline{\qquad\quad -ac^2\qquad\ -\ \ abc+a^2c\qquad -bc^2+b^2c+c^3} \\
a^3\qquad\qquad\qquad\ \ -3\,abc\qquad\ +b^3\qquad\qquad\ +c^3
\end{array}$$

18. x^2-x-19. **19.** The terms $=12\,x^3-22\,x^3+35\,x^3-4\,x^3=21\,x^3$.

21. $x^2 + 2x + 1$. **24.** On multiplying the terms all cancel.

26. $ab - c = (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2) - x^4 + y^4 = x^4 + 4y^4 - x^4 + y^4$.

31. Dividend = divisor $\times$ quotient + remainder $= x^4 - 4x^2 + 12x - 9 + 9$.

32. $a + 2b - c = x^2 - 3x + 2 + 6x^2 - 20x + 16 - 4x^2 + 9x - 2 = 3x^2 - 14x + 16$. Then $(a + 2b - c) \div (x - 2) = 3x - 8$.

34. On dividing the remainder is -9, so that 9 must be added.

35.

$$
\begin{array}{l}
1 - x - x^2 \,\big)\, 1 + x + x^2 \,\big(\, 1 + 2x + 4x^2 + 6x^3 \\
\quad 1 - x - x^2 \\
\quad\quad 2x + 2x^2 \\
\quad\quad 2x - 2x^2 - 2x^3 \\
\quad\quad\quad 4x^2 + 2x^3 \\
\quad\quad\quad 4x^2 - 4x^3 - 4x^4 \\
\quad\quad\quad\quad + 6x^3 + 4x^4
\end{array}
$$

39.

$$
\begin{array}{l}
x + a \\
x + b \\
\hline
x^2 + ax \\
\quad + bx + ab \\
\hline
x^2 + x(a + b) + ab \\
x \ + c \\
\hline
x^3 + x^2(a + b) + abx \\
\quad + x^2(c \quad) + (a + b)cx + abc \\
\hline
x^3 + x^2(a + b + c) + x(ab + bc + ca) + abc
\end{array}
$$

Put $a = 1$, $b = 2$, $c = 3$ in the product and $(x + 1)(x + 2)(x + 3) = x^3 + 6x^2 + 11x + 6$; $(x - 1)(x - 3)(x + 4) = x^3 - 13x + 24$.

42. First side $= (1 + 2x + x^2)(1 + y^2) - (1 + x^2)(1 + 2y + y^2) = 1 + 2x + x^2 + y^2 + 2xy^2 + x^2y^2 - 1 - 2y - y^2 - x^2 - 2x^2y - x^2y^2 = 2x - 2y - 2x^2y + 2xy^2$.

43. $p^2 + 4 = x^2 - 2 + \frac{1}{x^2} + 4 = x^2 + 2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2$,

$$\therefore p^2(p^2 + 4) = \left(x - \frac{1}{x}\right)^2\left(x + \frac{1}{x}\right)^2 = \left(x^2 - \frac{1}{x^2}\right)^2 = q^2.$$

44.

$$
\begin{array}{l}
a + b + c \,\big)\, a^3 \qquad\qquad -3abc + b^3 + c^3 \,\big(\, a^2 - ab - ac + b^2 - bc + c^2 \\
\quad a^3 + a^2b + a^2c \\
\hline
\quad -a^2b - a^2c \qquad -3abc + b^3 + c^3 \\
\quad -a^2b \qquad -ab^2 - abc \\
\hline
\quad\quad -a^2c + ab^2 - 2abc + b^3 + c^3 \\
\quad\quad -a^2c \qquad - abc \qquad -ac^2 \\
\hline
\quad\quad\quad ab^2 - abc + ac^2 + b^3 + c^3 \\
\quad\quad\quad ab^2 \qquad\qquad + b^3 \quad + b^2c \\
\hline
\quad\quad\quad\quad -abc + ac^2 \qquad + c^3 - b^2c \\
\quad\quad\quad\quad -abc \qquad\qquad\quad - b^2c - bc^2 \\
\hline
\quad\quad\quad\quad\quad ac^2 \qquad + c^3 \qquad + bc^2 \\
\quad\quad\quad\quad\quad ac^2 \qquad + c^3 \qquad + bc^2 \\
\hline
\end{array}
$$

50.
$$\begin{array}{l|l|l} a-b & a^2(b-c)+a(c^2-b^2)+b^2c-bc^2 & a(b-c)+c^2-bc \\ & \underline{a^2(b-c)-a(b^2-bc)} & \\ & a(c^2-bc)+b^2c-bc^2 & \\ & \underline{a(c^2-bc)+b^2c-bc^2} & \end{array}$$

51. Dividend $=(x^2+1)(3x^3-10x^2-6)+2x^2-7x+6.$

52.
$$\begin{array}{l|l|l} x-y+2 & x^3 \qquad\qquad -y^3+6y^2-12y+8 & x^2+xy-2x+y^2-4y+4 \\ & \underline{x^3-x^2y+2x^2} & \\ & x^2y-2x^2 & \\ & \underline{x^2y \qquad -xy^2+2xy} & \\ & -2x^2+xy^2-2xy & \\ & \underline{-2x^2 \qquad +2xy-4x} & \\ & xy^2-4xy+4x-y^3+6y^2 & \\ & \underline{xy^2 \qquad\qquad -y^3+2y^2} & \\ & -4xy+4x \qquad +4y^2-12y & \\ & \underline{-4xy \qquad\qquad +4y^2-8y} & \\ & 4x \qquad\qquad -4y+8 & \\ & \underline{4x \qquad\qquad -4y+8} & \end{array}$$

Exercise 67 — Page 129

1. $3(x-9)$. **2.** $2(a-3)$. **3.** $a(a-3)$. **4.** $b(b-5)$.
5. $3a(a-5b)$. **6.** $6xy(x-2y)$. **7.** $(x+y)(a+b)$.
8. $(m-n)(p+1)$. **9.** $(a-b)(x-2y)$.

10. $x(a+b)+y(a+b)=(a+b)(x+y)$; $a(x+y)+b(x+y)=(x+y)(a+b)$.

11. $m(a-b)+n(a-b)=(a-b)(m+n)$; $a(m+n)-b(m+n)=(m+n)(a-b)$.

12. $x(x-a)+b(x-a)=(x-a)(x+b)$; $x(x+b)-a(x+b)=(x+b)(x-a)$.

13. $b(x+a)-x(x+a)=(x+a)(b-x)$; $x(b-x)+a(b-x)=(b-x)(x+a)$. **14.** $(a-b)(2c+3d)$. **15.** $(x+1)(x^2+1)$.
16. $(a-1)(a^2-3)$. **17.** $(x+1)(1-y)$. **18.** $(x+4)(x^2-3)$.
19. $(a-7)(a^2-4)$. **20.** $(x+1)(x^4-x^2+1)$.

23. $5x(2x-y)-3z(2x-y)=(2x-y)(5x-3z)$.
$a^2b(a+c)-3ab^2(a+c)=(a+c)(a^2b-3ab^2)=ab(a+c)(a-3b)$.

25. $8a(6x-7y)+5b(6x-7y)=(6x-7y)(8a+5b)$.

26. $(x+y)^2+4(x+y)=(x+y)(x+y+4)$; $2(a-b)^2-(a-b)=(a-b)(2a-2b-1)$.

Exercise 68 — Page 131

29. $a+b-c$. **30.** $m+n+p$. **31.** $a-b+c$. **32.** $x+2y+z$. **33.** $2a-b-3c$. **34.** $9x^2-6xy+y^2+x^2-6xy+9y^2+4x^2+12xy+9y^2$.

36. $x^4+2x^3+3x^2+2x+1+x^4-2x^3+3x^2-2x+1$.

38. $9x^2+4y^2+z^2-12xy+6xz-4yz-(x^2+4y^2+9z^2-4xy+6xz-12yz)$.

39. $x^2-10x+25$. **40.** $4x^2+20xy+25y^2$. **41.** $a^2+4ab+4b^2$. **42.** $4m^2-12mn+9n^2$. **43.** $a^2+9b^2+c^2-6ab-2ac+6bc$. **44.** $9x^2+y^2+4z^2-6xy-12xz+4yz$.

47. If the numbers are x and $x+1$, sum of squares $=x^2+(x+1)^2=2x^2+2x+1$. Square of the sum $=4x^2+4x+1$.

49. Squaring, $x^2+2+\frac{1}{x^2}=16$, then $x^2+\frac{1}{x^2}=14$.

50. $a^2x^2+2abxy+b^2y^2+b^2x^2-2abxy+a^2y^2+c^2(x^2+y^2)$
$=a^2(x^2+y^2)+b^2(x^2+y^2)+c^2(x^2+y^2)=(x^2+y^2)(a^2+b^2+c^2)$.

52. $x+y+z=0$, then the given expression is 0, as it is the square of $x+y+z$.

Exercise 69 — Page 134

1. $4a^2-9$. **2.** $16x^2-1$. **3.** x^2y^2-25. **4.** $a^2b^2-c^2$. **5.** $4m^4-9n^2$. **6.** $a^2b^2c^2-x^2y^2$. **7.** $x^2-\frac{1}{4}$. **8.** x^4-y^4. **9.** $(x+y)^2-z^2=x^2+2xy+y^2-z^2$. **10.** $(a-b)^2-c^2=a^2-2ab+b^2-c^2$. **11.** $\{a+(b-c)\}\{a-(b-c)\}=a^2-(b-c)^2$. **12.** $(2x+3y)^2-25$. **13.** $p^2-(2q-3r)^2$. **14.** $(1+x^2)^2-x^2$. **15.** $(a-c)^2-(b+d)^2$. **16.** $(a-2b)^2-(c-2d)^2$. **17.** $(x-3)(x+3)$. **18.** $(2x-5)(2x+5)$. **19.** $(a-2b)(a+2b)$. **20.** $(ab-x)(ab+x)$. **21.** $(4x-3y)(4x+3y)$. **22.** $(93-4)(93+4)=89\times 97$. **23.** $(1-ab)(1+ab)$. **24.** $(5-x^2)(5+x^2)$. **25.** $(a-b+c)(a-b-c)$. **26.** $(x+y-5)(x+y+5)$. **27.** $(c+a+b)(c-a-b)$. **28.** $(x-y+z)(x+y-z)$. **29.** $(a+b-c+d)(a+b+c-d)$. **30.** $(a+2b-2c)(a+2b+2c)$. **31.** $(x+y-a)(x+y+a)$. **32.** $(a-b+c)(a-b-c)$. **33.** $a^2-(b+c)^2=(a-b-c)(a+b+c)$. **34.** $a^2-(b-2c)^2=(a+b-2c)(a-b+2c)$. **35.** $(4x+3+4x)(4x+3-4x)=3(8x+3)$. **36.** $1-(x-y)^2=(1-x+y)(1+x-y)$. **37.** $(a+y)^2-x^2=(a+y+x)(a+y-x)$. **38.** $(a+b)^2-(c+d)^2=(a+b+c+d)(a+b-c-d)$. **39.** $(a-c)^2-(b+d)^2=(a-c+b+d)(a-c-b-d)$. **40.** $(a-1)^2-(b-c)^2=(a-1-b+c)(a-1+b-c)$.

41. $(x^2-y^2)^2-(x+2)^2=(x^2-y^2+x+2)(x^2-y^2-x-2)$.

42. $(2x-1)^2-(y-2a)^2=(2x-1+y-2a)(2x-1-y+2a)$.

43. $(1-a)^2-(b-2c)^2=(1-a+b-2c)(1-a-b+2c)$.

50. $x^4 - a^2x^2 - y^4 + a^2y^2 + 2\,xy(x^2-y^2) = (x^2-y^2)(x^2+y^2) - a^2(x^2-y^2) + 2\,xy(x^2-y^2) = (x^2-y^2)(x^2+y^2-a^2+2\,xy)$.

53. $(x-y)(y^2-z^2)-(x^2-y^2)(y-z) = (x-y)(y-z)(y+z-x-y)$.

54. $(b-c)(ab+ac-bc-a^2) = (b-c)\{a(c-a)-b(c-a)\} = (b-c)(c-a)(a-b)$.

Exercise 70 — Page 136

1. $(a^2+1)^2-a^2 = (a^2+a+1)(a^2-a+1)$. **2.** $(x^2+5)^2-9\,x^2 = (x^2+3\,x+5)(x^2-3\,x+5)$. **3.** $(x^2+4)^2-x^2 = (x^2+x+4)(x^2-x+4)$. **4.** $(x^2+3\,y^2)^2-4\,x^2y^2 = (x^2+2\,xy+3\,y^2)(x^2-2\,xy+3\,y^2)$. **5.** $(2\,a^2+1)^2-4\,a^2 = (2\,a^2+2\,a+1)(2\,a^2-2\,a+1)$. **6.** $(3\,x^2+4\,y^2)^2-16\,x^2y^2 = (3\,x^2+4\,xy+4\,y^2)(3\,x^2-4\,xy+4\,y^2)$. **7.** $(2\,b^2-1)^2-9\,b^2 = (2\,b^2+3\,b-1)(2\,b^2-3\,b-1)$. **8.** $(3\,a^2-1)^2-9\,a^2 = (3\,a^2+3\,a-1)(3\,a^2-3\,a-1)$. **9.** $(3\,a^2-8\,b^2)^2-4\,a^2b^2 = (3\,a^2+2\,ab-8\,b^2)(3\,a^2-2\,ab-8\,b^2)$. **10.** $(5\,x^2-8\,y^2)^2-9\,x^2y^2 = (5\,x^2-3\,xy-8\,y^2)(5\,x^2+3\,xy-8\,y^2)$. **11.** $(x^2-y^2)^2-9\,x^2y^2 = (x^2+3\,xy-y^2)(x^2-3\,xy-y^2)$.

12. $(x^4+1)^2-9\,x^4 = (x^4+3\,x^2+1)(x^4-3\,x^2+1)$.

16.
$$\begin{aligned}(a^2-b^2+c^2)^2-4\,a^2c^2 &= (a^2-b^2+c^2+2\,ac)(a^2-b^2+c^2-2\,ac)\\ &= \{(a+c)^2-b^2\}\{(a-c)^2-b^2\}\\ &= (a+c+b)(a+c-b)(a-c+b)(a-c-b).\end{aligned}$$

17.
$$\begin{aligned}&\{(a+1)^2+(a-1)^2\}^2-(a^2-1)^2\\ &= \{(a+1)^2+(a-1)^2+a^2-1\}\{(a+1)^2+(a-1)^2-a^2+1\}.\end{aligned}$$

Exercise 71 — Page 139

1. $(x+1)(x+3)$. **2.** $(a+5)(a+6)$. **3.** $(y+5)(y+3)$. **4.** $(a-9)(a-2)$. **5.** $(x-6)(x-8)$. **6.** $(1+2\,x)(1+3\,x)$. **7.** $(x-1)(x-14)$. **8.** $(ab-2)(ab-3)$. **9.** $(a-7)(a-8)$. **10.** $(1-2\,x)(1-19\,x)$. **11.** $(x-2\,y)(x-4\,y)$. **12.** $(a-4\,b)(a-9\,b)$. **13.** $(x-5)(x+1)$. **14.** $(a+2)(a-11)$. **15.** $(x+1)(x-29)$. **16.** $(y-7)(y+3)$. **17.** $(1-5\,a)(1+3\,a)$. **18.** $(a-2\,y)(a+y)$. **19.** $(x+1)(2\,x+3)$. **20.** $(2\,x+y)(2\,x+3\,y)$. **21.** $(3\,a-2\,b)(3\,a-4\,b)$. **22.** $(x-1)(8\,x+9)$. **23.** $(x-1)(3\,x+2)$. **24.** $(2\,a+1)(3\,a-2)$. **25.** $(x-1)(4\,x+5)$. **26.** $(3\,b+1)(5\,b-8)$.

27. $(5\,x+1)(2\,x-5)$. **28.** $(b-9)(10\,b+1)$.

29. $(x-3\,y)(9\,x-4\,y)$. **30.** $(5\,a-2\,b)(2\,a-5\,b)$.

37. $(x^2+4\,x-5)(x^2+4\,x+3) = (x-1)(x+5)(x+1)(x+3)$.

38. $(x^2-9\,x-10)(x^2-9\,x+14) = (x+1)(x-10)(x-2)(x-7)$.

39. Factor each side and it becomes an identity.

41.
$$\begin{aligned}&(3\,x-2)(2\,x-3)\times(x-1)(2\,x-5)\div(x-1)(3\,x-2)\\ &= (2\,x-3)(2\,x-5).\end{aligned}$$

42. If the factors are $(3x-14)(x+1)$, then $a=-11$; if $(3x+7)(x-2)$, then $a=1$; if $(3x-7)(x+2)$, then $a=-1$, etc.

43. $(3a-4b)(2a+5b)(2a-7b)(11a+2b)\div(2a+5b)(2a-7b)$.

44. $(x+4y)(x+y)+(x+y)=(x+y)(x+4y+1)$.

45. $(3a+2b)(a-b)+2(3a+2b)=(3a+2b)(a-b+2)$.

46. $(x+1)(x+2)(x-1)(x+1)(x^2+1)\div(x+1)^2(x+2)(x-1)=x^2+1$.

Exercise 72 — Page 141

1. $a+b$. **2.** $x+2$. **3.** $x-3$. **4.** $10-a$. **5.** $x-4y$.
6. $3-b$. **7.** $2a+5$. **8.** $5a-2b$. **9.** $1-3x$. **10.** $7x-2$.
11. $a+b+c$. **12.** $a-b-c$. **13.** $(a+3)(a^2-3a+9)$.
14. $(x-2y)(x^2+2xy+4y^2)$. **15.** $(2a+1)(4a^2-2a+1)$.
16. $(3x-4y)(9x^2+12xy+16y^2)$. **17.** $(2-3a)(4+6a+9a^2)$.
18. $(10x-y)(100x^2+10xy+y^2)$. **19.** $(a+b^2)(a^2-ab^2+b^4)$.
20. $(x^2-b)(x^4+bx^2+b^2)$. **21.** $(a-y^2)(a^2+ay^2+y^4)$.
22. $2(a^3-8)=2(a-2)(a^2+2a+4)$.

32. One factor is $2a-3b+3a-2b$ or $5(a-b)$.

34. $(a^6-b^6)(a^6+b^6)=(a^3-b^3)(a^3+b^3)(a^2+b^2)(a^4-a^2b^2+b^4)$
$=(a-b)(a^2+ab+b^2)(a+b)(a^2-ab+b^2)(a^2+b^2)(a^4-a^2b^2+b^4)$.

35. One factor is $2x^2-3x+3-x^2+2x-5=(x+1)(x-2)$.

36. $\left(x+\frac{1}{x}\right)^3=8$ or $x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=8$ or $x^3+\frac{1}{x^3}=2$.

37. The first side $=(a+b)^2(a^2-ab+b^2)=(a+b)(a^3+b^3)$.

Exercise 73 — Page 143

1. $(x-1)(x-4)(x-5)$. **2.** $(x-1)(x^2-2x-14)$.
3. $(x-2)(x+3)(x+4)$. **4.** $(x+1)(x-2)(x-3)$.
5. $(x-1)(x-2)(2x-1)$. **6.** $(x+1)(x-3)(4x-1)$.

16. If $x=-2$, then $x^3-10x+a=-8+20+a=0$ or $a=-12$.

20. If $x-1$ is a factor, $1-5+a+b=0$ or $a+b=4$. Also if $x-2$ is a factor, $8-20+2a+b=0$ or $2a+b=12$. Solving, $a=8$, $b=-4$.

21. If $x-2$ is a factor, $8p-12+2q-10=0$ or $8p+2q=22$. Also $8q+8-34+p=0$ or $p+8q=26$. Solving, $p=2$, $q=3$.

Exercise 74 — Page 145

11. $x^2-5x+6=0$. **12.** $x^2+x-20=0$. **13.** $x^2+6x+8=0$.
14. $x^2-x(a+b)+ab=0$. **15.** $(x-2)(x-3)(x-1)=0$.
16. $(x-4)(x-5)(x+6)=0$. **17.** 5, 3. **18.** -5, -3.
19. 3, -5. **20.** 5, -3. **21.** 2, $\frac{2}{3}$. **22.** 1, $-\frac{1}{2}$.
23. -3, $2\frac{1}{2}$. **24.** 2, $-1\frac{2}{3}$. **25.** 0, 1, -1. **26.** a, $-b$.

27. Divide by $x-2$ and factor the quotient. The roots are 2, 3, -5.

28. $(x-1)(x-2)(x-3)=0$; $(x-1)(2x-1)(2x-3)=0$.

29. $x^2+x=42$ or $(x-6)(x+7)=0$, then $x=6$ or -7.

30. $x^2+(x+1)^2=61$ or $x^2+x-30=0$, $x=5$ or -6, $x+1=6$ or -5.

31. $(x+2)^2=x^2+(x+1)^2$ or $x^2-2x-3=0$, then $x=3$.

Exercise 75 — Page 147

4. $(x+a)^2-b^2+(x-a)^2-b^2$. **5.** $(x^2+x)^2-1-(x^2-x)^2+1$.

8. $9999+9998$. **9.** $(5743+4257)(5743-4257)=10000\times 1486$.

10. $500^2-3^2-500^2+2^2$. **11.** $a+99+a+98=2a+197$.

12. $(x-7)(x+6)$. **13.** $(x-1)(x+1)(x-3)$. **14.** $x(x-2)(x+2)$. **15.** $(a-7)(a+8)$. **16.** $(x-1)(x+a)$. **17.** $3(3x-2y)(3x+2y)$. **18.** $(x+5)(x+2)(x-2)$. **19.** $x(x-1)(x-2)$. **20.** $(3a+1)(5a+9)$. **21.** $(7-x)(49+7x+x^2)$. **22.** $x^2(x-2)(x+2)$. **23.** $2(3x+4)^2$. **24.** $(x-3)(2x+1)$. **25.** $(3x-5y)(5x+3y)$. **26.** $(1+a-b)(1-a+b)$. **27.** $(x-2)(x+y-1)$. **28.** $(ac+bd)(ad+bc)$. **29.** $x^2y(5x-4y)^2$.

34. $4(27a^3-125)=4(3a-5)(9a^2+15a+25)$.

35. $x^2-y^2+x+y=(x+y)(x-y)+(x+y)=(x+y)(x-y+1)$.

37. $(x-y)(x^2+xy+y^2)-2xy(x-y)=(x-y)(x^2-xy+y^2)$.

39. $a^2-c^2-2ab+b^2=(a-b)^2-c^2=(a-b+c)(a-b-c)$.

40. $(x+y)(x^2-xy+y^2)+3xy(x+y)=(x+y)(x^2+2xy+y^2)=(x+y)^3$. **41.** $ax(x-3b)-2(x-3b)=(x-3b)(ax-2)$.

42. $(a-2b)(a+2b)-3(a+2b)=(a+2b)(a-2b-3)$.

43. $4x^2+2ax-y^2-ay=(2x+y)(2x-y)+a(2x-y)$.

44. $(a+b)^2+c(a+b)$. **45.** $(a-b)^2-(a-b)$.

46. $(x^3-y^3)+(x^2-y^2)+(x-y)=(x-y)(x^2+xy+y^2+x+y+1)$.

48. $(2a-5b)(2a+5b)+(2a+5b)=(2a+5b)(2a-5b+1)$.

49. $(2a+2b)^3-(2a-b)^3$
$=(2a+2b-2a+b)\{(2a+2b)^2+(2a+2b)(2a-b)+(2a-b)^2\}$
$=3b(12a^2+6ab+3b^2)=9b(4a^2+2ab+b^2)$.

50. $(x^2-y^2)^2-16x^2y^2$. **51.** $(a^2-b^2)^2-(a-3)^2$.

56. Since $x+y$ is a factor of x^3+y^3 then one factor of the given expression is $2a-3b+4c+2a-b$ or $4(a-b+c)$.

62. $(x^2-5x+4)(x^2-5x+6)=(x-1)(x-4)(x-2)(x-3)$.

64. If they are x and $x+6$, then $(x+6)^2-x^2=12x+36=6(2x+6)$.

65. $(x-b)(x+c)\times(x-c)(x+a)\div(x+a)(x-b)=(x+c)(x-c)$.

66. $a^2-b^2-c^2+2bc=a^2-(b-c)^2=(a+b-c)(a-b+c)$.

67. Putting $x=1$, $1+1+a+b-3=0$ or $a+b=1$. Putting $x=-3$, $81-27+9a-3b-3=0$ or $9a-3b=-51$. Solving, $a=-4$, $b=5$.

68. $(2x^2-ax-a^2)+(bx-ab)=(x-a)(2x+a)+b(x-a)$.

70. See Ex. 16, page 137.

71. $(a^2-b^2-c^2+d^2+2ad-2bc)(a^2-b^2-c^2+d^2-2ad+2bc)$
$=\{(a+d)^2-(b+c)^2\}\{(a-d)^2-(b-c)^2\}$
$=(a+d+b+c)(a+d-b-c)(a-d+b-c)(a-d-b+c)$.

Exercise 76 — Page 151

1. $x=5-y$, $y=5-x$. **2.** $x=3+y$, $y=x-3$.

3. $x=11-2y$, $y=\frac{1}{2}(11-x)$. **4.** $x=\frac{1}{3}(6+y)$, $y=3x-6$.

5. $x=\frac{1}{2}(12-3y)$, $y=\frac{1}{3}(12-2x)$. **6.** $x=\frac{1}{5}(19+4y)$, $y=\frac{1}{4}(5x-19)$.

7. From (1) $x=18-2y$. Substitute in (2) and $2(18-2y)+5y=41$, $\therefore y=5$ and $x=8$. **8.** $x=2$, $y=1$. **9.** $x=11$, $y=3$.

10. $x=10$, $y=2$. **11.** $x=4$, $y=\frac{1}{2}$. **12.** $x=\frac{1}{2}$, $y=\frac{1}{3}$.

13. From (1) $x=10-3y$, from (2) $x=14-5y$, $\therefore 10-3y=14-5y$, $\therefore y=2$ and $x=4$.

14. $y=26-2x$ and $y=3x-14$, $\therefore 26-2x=3x-14$; $x=8$, $y=10$.

15. $x=\frac{1}{3}(10-4y)=\frac{1}{4}(5+3y)$, $\therefore 40-16y=15+9y$; $y=1$, $x=2$.

16. $5x-3y=30$, $10x+9y=135$; $x=9$, $y=5$.

17. $3x-2y=0$, $3x-4y=-12$; $x=4$, $y=6$.

18. $x-2y=-12$, $10x-3y=50$; $x=8$, $y=10$.

19. $3x-2y=0$, $5x-12y=-156$; $x=12$, $y=18$.

20. $9x+5y=90$, $4x+3y=47$; $x=5$, $y=9$.

21. $8x-3y=0$, $3x-y=1$; $x=3$, $y=8$.

22. $33x+22-y-7=110$, or $33x-y=95$; $14y+x+11=70$, or $x+14y=59$; $x=3$, $y=4$.

23. $x-5y+3=2x-8y+3$, or $-x+3y=0$; $x-5y+3=7x-10y+16$, or $-6x+5y=13$; $x=-3$, $y=-1$.

24. $xy-y-2x+2-xy+3x-y+3=17$ or $x-2y=12$, $xy-3y-5x+15-xy+5y+3x-15=-22$ or $-2x+2y=-22$; $x=10$, $y=-1$. **25.** $10x+21y=-52$, $x+y=-300$; $x=-568$, $y=268$.

26. $x+5=3y-9$ or $x-3y=-14$, and $45x-36+99y=22x-55+1801$ or $23x+99y=1862$; $x=25$, $y=13$.

27. Equating the first two, $144x - 30y = 62$. Equating the last two, $72x + 30y = 46$; $x = \frac{1}{2}$, $y = \frac{1}{3}$.

28. If $x + y = \frac{4}{3}x$, then $x = 3y$. $\therefore x - y = 2y$ or $\dfrac{x-y}{y} = 2$.

Exercise 77 — Page 153

5. Eliminate x from (1) and (2) and we get $y + z = 8$. Eliminate x from (2) and (3) and we get $y + 6z = 17$. Solving for y and z, we get $y = 6\frac{1}{5}$, $z = 1\frac{4}{5}$, and then $x = -1\frac{4}{5}$. To verify substitute these values in each of the given equations.

6. From (1) and (2), $5x - z = 14$. From (2) and (3), $9x + z = 28$. $\therefore x = 3$, $z = 1$, $y = 2$.

7. From (1) and (2), $2x - y = 3$. From (1) and (3), $3x - 4y = 2$. $\therefore x = 2$, $y = 1$, $z = 5$.

8. From (1) and (2), $2x = 20$ or $x = 10$. From (1) and (3), $3x + 3y = 54$. $\therefore x = 10$, $y = 8$, $z = 2$.

9. From (1) and (2), $-2x + 9y = 37$. From (1) and (3), $10x + 14y = 110$. $\therefore x = 4$, $y = 5$, $z = 6$.

10. Add all three and divide by 2 and $x + y + z = 85$; $x = 10$, $y = 15$, $z = 60$.

11. Eliminate x from (1) and (3) and $4y + 5z = 3$. Solving $4y + 5z = 3$, $3y + 4z = 2$; $x = 8$, $y = 2$, $z = -1$.

12. The equations become $3z - 2y = 1$, $4x + 4y - 9z = -4$, $35x - 2y - 21z = -9$. Eliminate y from (1) and (2) and from (1) and (3) and we get $4x - 3z = -2$, $35x - 24z = -10$. Solving, $x = 2$, $z = 3\frac{1}{3}$, $y = 4\frac{1}{2}$.

13. Eliminate x from (1) and (2) and $y + z = -2$; $x = 2$, $y = 0$, $z = -2$.

14. Removing fractions $2x + 4y + 3z = 432$, $20x + 12y + 9z = 1800$, $5x + 10y + 2z = 860$. From (1) and (2) $28y + 21z = 2520$ or $4y + 3z = 360$. From (2) and (3) $28y - z = 1640$. $\therefore y = 60$, $z = 40$, $x = 36$.

15. Put each $= 1$, remove fractions, and solve. $x = 12$, $y = -60$, $z = 60$.

16. Solving $x = 5$, $y = 10$, $z = 15$, $\therefore x + y + z = 30$.

17. Solve the first three and $x = 6\frac{1}{2}$, $y = -5$, $z = -2\frac{1}{2}$, $\therefore w = 13$.

18. $a - b + c = 8$, $4a - 2b + c = 8$, $9a - 3b + c = 10$; $a = 1$, $b = 3$, $c = 10$.

19. $a + b + c = 9$, $a - b + c = -3$, $4a + 2b + c = 18$; $a = 1$, $b = 6$, $c = 2$, and when $x = 3$, $ax^2 + bx + c = 29$.

20. If they are x, y, z, then $x + y + z = 9$, $x + 2y + 3z = 22$, $x + 4y + 9z = 58$. Solving, $x = 1$, $y = 3$, $z = 5$.

21. Add (1) and (3) and subtract (2) and $a + d = 16$.

Exercise 78 — Page 155

1. Give y any value, say 4, then $x = 12$. Therefore 12, 4 is one pair. Similarly 6, 0; 0, -4; 5, $-\frac{2}{3}$ are others.

2. If $2x + 3y = 13$, $5x - y = 24$, then $x = 5$, $y = 1$. $\therefore 4x + 5y$ would be 25, not 19. It is therefore impossible.

3. They are impossible. They are indeterminate and dependent.

4. If we eliminate z from (1) and (2), the result is (3). They are therefore indeterminate. If $z = 5$, then $x = 3$, $y = 4$.

5. Put $x = 3$, then $y + z = 7$, $2y + z = 2$, $\therefore y = -5$, $z = 12$. One solution therefore is 3, -5, 12. Similarly others may be found.

6. Solving (1) and (2), $x = 5$, $y = 10$. $\therefore x + 4y = 45$, $\therefore a = 45$.

7. Solving (1) and (2), $x = 1$, $y = 2$. $\therefore 3 + 2a = 11$, $\therefore a = 4$.

8. Solving (2) and (3), $x = -\frac{1}{3}$, $y = -3$. $\therefore -3 + 3a = 0$. $\therefore a = 1$.

9. Eliminate x from (1) and (3) and $y + z = -8$. Eliminate x from (2) and (3), then $y + z = -13$. These results are inconsistent.

Exercise 79 — Page 156

1. Multiply (1) by 3 and add and $\frac{42}{x} = 7$; $x = 6$, $y = 7$.

2. Multiply (1) by 6 and (2) by 7 and add and $\frac{47}{x} = 188$; $x = \frac{1}{4}$, $y = \frac{1}{3}$.

3. Eliminate y and $\frac{62}{x} = 186$; $x = \frac{1}{3}$, $y = \frac{1}{2}$.

4. Eliminate y and $\frac{115}{x} = 460$; $x = \frac{1}{4}$, $y = \frac{1}{7}$.

5. Eliminate x and $19y = 57$; $y = 3$, $x = \frac{1}{3}$.

6. Eliminate x from (1) and (3) and $\frac{2}{y} + \frac{4}{z} = 28$. Solve, $\frac{3}{y} - \frac{2}{z} = 2$, $\frac{2}{y} + \frac{4}{z} = 28$; $y = \frac{1}{4}$, $z = \frac{1}{5}$, $x = \frac{1}{3}$.

7. Divide each equation by xy and solve as before; $x = \frac{1}{7}$, $y = \frac{1}{4}$.

8. $\frac{18}{x} + \frac{3}{y} = 11$, $\frac{45}{x} + \frac{2}{y} = 11$; $x = 9$, $y = \frac{1}{3}$.

9. Put each $= 30$ and solve as before; $x = \frac{1}{3}$, $y = \frac{1}{5}$.

10. Equate the first two and $-9x - \frac{3}{y} = 15$. Equate the last two and $14x + \frac{4}{y} = -28$. Solving, $x = -4$, $y = \frac{1}{7}$.

11. Equate each to $8z+17$ and we get $\frac{3}{x}+\frac{2}{y}-12z=17$, $\frac{1}{x}+\frac{4}{y}-8z=17$, $-\frac{1}{x}+\frac{5}{y}+4z=17$. Eliminate x from (1) and (2) and from (2) and (3), then $\frac{10}{y}-12z=34$, $\frac{9}{y}-4z=34$. Then $y=\frac{1}{4}$, $z=\frac{1}{2}$, $x=\frac{1}{5}$.

Exercise 80 — Page 158

1. If $\$x$ and $\$y$ are the wages, $10x+4y=7x+10y=96$; $x=8$, $y=4$.

2. Let the nos. be $5x$ and $7x$, then $7x-5x=10$.

3. $x+y+z=370$, $x+y-z=70$, $6x=4z$; $x=100$, $y=120$, $z=150$.

4. $x+y=29$, $x+z=33$, $y+z=36$. Add and divide by 2, then $x+y+z=49$. $\therefore x=13$, $y=16$, $z=20$.

5. Let the nos. be $7x$, $4x$, $2x$, then $7x+4x+2x=429$.

6. Suppose his wages were $\$x$ and expenses $\$y$, then $x-y=200$ and $\frac{11}{10}x-\frac{19}{20}y=310$. Solving, $x=800$, $y=600$.

7. Let $\frac{x}{x+3}$ be the fraction, then $\frac{x-2}{x+1}=\frac{5}{6}$; $x=17$.

8. $x+y+z=120$, $\frac{3}{4}x-y=5$, $\frac{3}{4}y-z=10$; $x=60$, $y=40$, $z=20$.

9. If the wages are $\$x$ and $\$y$, then $6x+2y=28$, $7x+5y=38$. $\therefore x=4$, $y=2$. $\therefore$ 3 men and 4 boys earn $20 per day.

10. If the units digit is x and the tens y, the no. $=10y+x$ and the reversed no. $=10x+y$. $\therefore 10y+x=8(x+y)$, $10y+x-45=10x+y$. Solving, $x=2$, $y=7$. $\therefore$ no. $=72$.

11. The units digit being x and the tens y, then $x-y=4$, $10y+x+10x+y=110$. $\therefore y=3$, $y=7$. $\therefore$ no. $=73$ or 37.

12. The units digit being x and the tens y, $x+y=14$, $10y+x+18=10x+y$. $\therefore x=8$, $y=6$. $\therefore$ no. $=68$.

13. If $\frac{x}{y}$ is the fraction, $\frac{x+1}{y+1}=\frac{1}{3}$, $\frac{x}{y-9}=\frac{1}{2}$; $x=7$, $y=23$.

14. If x is the units digit and y the tens, $x+y=11$, $10x+y=2(10y+x)+7$. $\therefore x=8$, $y=3$. $\therefore$ no. $=38$.

15. If x is the units digit, then the tens is $x+6$. $\therefore$ the no. $=10(x+6)+x$ or $11x+60$. The reversed no. $=10x+x+6$ or $11x+6$. $\therefore$ the difference $=54$.

16. $\frac{1}{x}+\frac{1}{y}=\frac{9}{20}$, $\frac{6}{x}-\frac{5}{y}=\frac{1}{2}$; $x=4$, $y=5$.

17. If x is the less, then $150-x$ is the greater, then

$$\frac{150-x}{x}=3+\frac{2}{x};\ x=37.$$

18. If I take x lb. of the first and y of the second, $x + y = 100$, $30x + 40y = 3400$; $x = 60$, $y = 40$.

19. If they cost x¢ and y¢, then $3x + 10y = 240$, $\frac{11}{10} \times 3x + \frac{9}{10} \times 10y = 252$; $x = 60$, $y = 6$.

20. Let the nos. be $7x$ and $5x$, then $36x \div 12x = 3$.

21. If x is the units digit and y the tens, then $(10x + y) + (10y + x) = 11(x + y)$ and is divisible by 11. Similarly for the diff.

22. If x is the units digits and y the hundreds, then the number is $100y + x$. $\therefore 100y + x + 396 = 100x + y$ or $99y - 99x = -396$, or $y - x = -4$, and $100y + x - 5(x + y) = 257$. $x = 7$, $y = 3$.

23. If the result is x, then the numbers are $x - 2$, $x + 2$, $\frac{1}{2}x$, $2x$. $\therefore x - 2 + x + 2 + \frac{1}{2}x + 2x = 126$; $x = 28$.

24. $x + 2y + 2z = 44$, $2x + y + 2z = 42$, $2x + 2y + z = 39$. Add and divide by 5 and $x + y + z = 25$ or $2x + 2y + 2z = 50$. Subtract each equation from this and $x = 6$, $y = 8$, $z = 11$.

25. Let the units be x, the tens y, the hundreds z, then $x + y + z = 12$, $100z + 10x + y = 100z + 10y + x + 36$ or $x - y = 4$. $100x + 10y + z = 100z + 10y + x + 198$ or $x - z = 2$. Then $x = 6$, $y = 2$, $z = 4$, and the no. $= 426$.

26. $x + \frac{1}{3}y + \frac{1}{3}z = 25$, $\frac{1}{4}x + y + \frac{1}{4}z = 25$, $\frac{1}{5}x + \frac{1}{5}y + z = 25$; $x = 13$, $y = 17$, $z = 19$.

27. If A takes x days, then he does $\frac{1}{x}$ in 1 day, and if B takes y days, then $\frac{6}{x} + \frac{21}{y} = 1$, $\frac{8}{x} + \frac{18}{y} = 1$; $x = 20$, $y = 30$.

28. If the first is x, the second is $\frac{3}{2}x$, the third is $\frac{4}{3}$ of $\frac{3}{2}x$ or $2x$, and the fourth is $\frac{5}{4}$ of $2x$ or $\frac{5}{2}x$. $\therefore x + \frac{3}{2}x + 2x + \frac{5}{2}x = 84$. $\therefore x = 12$.

29. $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$, $\frac{1}{x} + \frac{1}{z} = \frac{1}{3}$, $\frac{1}{y} + \frac{1}{z} = \frac{1}{4}$. Add all and divide by 2 and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{13}{24}$. Subtract each equation from this and $x = 3\frac{3}{7}$, $y = 4\frac{4}{5}$, $z = 24$.

30. If x be the units, y the tens, and z the hundreds, then $x + 10y + 100z + 100x + 10y + z = 1029$ or $101x + 20y + 101z = 1029$, $x + y + z = 15$ and $z - x = 5$; $x = 2$, $y = 6$, $z = 7$. $\therefore$ the nos. are 267 and 762.

31. Let x mi. per hour be his rate in still water, then his rate up stream is $x - 2$ and down is $x + 2$ mi. per hour. If y mi. is the distance, then $\frac{y}{x - 2} = 5$, $\frac{y}{x + 2} = 1\frac{2}{3}$; $x = 4$.

32. If x were sold at \$125, then $50-x$ were sold at \$150, therefore the total selling price was $125x+150(50-x)$ dollars. $\therefore 125x+150(50-x)-50=7200$; $\therefore x=10$.

33. Let the units digit be x, then the tenths is $x-1$, then the number is $x+\frac{1}{10}(x-1)$. $\therefore x+x-1=2x+\frac{1}{5}(x-1)-2$. $\therefore x=6$.

34. Let $x=$ no. of lb. of tea and y¢ $=$ price per lb. of coffee, then $60x+100y=12000$, $75x+120y=14800$; $\therefore x=133\frac{1}{3}$.

35. If the fraction is $\frac{x}{y}$, then $\frac{x-2}{y-2}=\frac{1}{2}$. $\therefore y=2x-2$. $\therefore \frac{x-1}{y}=\frac{x-1}{2x-2}=\frac{1}{2}$, which was to be shown.

36. $x+z=15$, $y+z=14$, $x+y=13$; $\therefore x=7$, $y=6$, $z=8$.

37. If $AB=10$, $BC=15$, $CA=19$, then $x=7$, $y=3$, $z=12$.

39. When $m=10$ and $n=15$, $\frac{mn}{m+n}=\frac{150}{25}=6$; when $m=20$ and $n=5$, it equals 4; when $m=\frac{3}{4}$ and $n=1\frac{1}{3}$, it equals $\frac{12}{25}$.

Exercise 81 — Page 161

1. 6, 4. **2.** $\frac{4}{5}, -\frac{3}{5}$. **3.** 1, 2. **4.** $\frac{1}{2}$, 2. **5.** 1, $1\frac{1}{4}$. **6.** 3, 4. (See Ex. 7, page 156.) **7.** $x-7y=-1$, $x-6y=6$; $x=48$, $y=7$. **8.** $x-2y=3$, $x-4y=-1$; $x=7$, $y=2$. **9.** $x-y=-12$, $2x+3y=-20$; $x=-11\frac{1}{5}$, $y=\frac{4}{5}$. **10.** $7x-14y=9x-3y$ or $2x+11y=0$; $x=3\frac{7}{27}$, $y=-\frac{16}{27}$. **11.** $15x-y=27$, $x+12y=38$; $x=2$, $y=3$.

12. $2(x-1)=y-3$, $3(x-1)=z-5$, $x+y+z=33$; $x=5$, $y=11$, $z=17$.

13. $x=-11$, $y=3$, $z=5$. **14.** $x=1$, $y=2$, $z=3$. **15.** $x=6$, $y=8$, $z=10$.

16. $12x-24-200+20x=15y-150$ or $32x-15y=74$, $16y+32-6x-3y=6x+78$ or $12x-13y=-46$; $x=7$, $y=10$.

17. $20x-20-5y-25=x+2$ or $19x-5y=47$, $xy-1\frac{1}{2}y-1\frac{1}{3}x+2=xy-5$ or $1\frac{1}{3}x+1\frac{1}{2}y=7$; $x=3$, $y=2$.

18. -1, 2, 1. **19.** 6, 2, 1. **20.** 3, 2, 1, 0. **21.** 2, 3. **22.** 4, 1.

23. $x-2y=3(x-10)$ or $2x+2y=30$, $x+y=5(x-10)$ or $4x-y=50$; $x=13$, $y=2$.

24. Eliminate x from (1) and (2) and $8x+9y=38$. Eliminate x from (1) and (3) and we get an equivalent equation.

25. If the parts are x and $1-x$, then $18x-12(1-x)=13$; $\therefore x=\frac{5}{6}$.

26. If x is the units and y the tens, $10x+y=4(x+y)$, $10x+y+18=10y+x$; $\therefore x=4$, $y=2$, and the no. $=24$.

27. Let x be the units and $2x$ the tens. The no. $= 20x + x$ or $21x$. $\therefore 21x \div 3x = 7$.

28. Let $\frac{x}{y}$ be the fraction, then $\frac{x}{y} = \frac{3}{4}$, $\frac{1}{3}y - \frac{2}{9}x = 8$; $x = 36$, $y = 48$.

29. Let their rates be x and y miles per hour, then $5(x + y) = 30$, $15(x - y) = 30$; $x = 4$, $y = 2$.

30. Let B's age be x years and C's y years, then A's is $(x + y)$ years. $\therefore x + y - 10 = 2(x - 10)$ or $y + 10 = x$. In 10 years A's age will be $x + y + 10$ years and C will be $y + 10$ years, and since $y + 10 = x$, A will then be twice as old as C.

31. Let x and y be the numbers, then $50x + 25y = 1950$ and $4y - 2x = 12$; $x = 30$, $y = 18$.

32. If the prices are x¢ and y¢, then $5x + 8y = 580$, $\frac{115}{100} \times 5x + \frac{11}{10} \times 8y = 653$; $x = 60$, $y = 35$.

33. Let the sums be \$$x$ and \$$y$, then $\frac{4}{100}x + \frac{6}{100}y = 42$ and $\frac{6}{100}x + \frac{4}{100}y = 50\frac{1}{2}$; $\therefore x = 675$, $y = 250$.

34. If the sides are x and y ft., then $(x + 5)(y + 5) = xy + 275$ and $(x - 5)(y - 5) = xy - 225$. When simplified, these two equations are equivalent, and therefore two unknowns cannot be found from them.

35. $7x + 3y - z = 10$, $7x + y = 9$, $4x - z = 1$; $x = 1$, $y = 2$, $z = 3$.

36. If he uses x and y lb., then $x + y = 60$, $30x + 40y = 2160$; $\therefore x = 24$, $y = 36$.

37. When $x = 1$, $a + b + c = 6$; when $x = 2$, $4a + 2b + c = 13$; when $x = 3$, $9a + 3b + c = 26$. Solve for a, b, c, and the result is $a = 3$, $b = -2$, $c = 5$.

38. Let x be the units, then $x + 3$ is the tens. The number $= 10(x + 3) + x$ or $11x + 30$. The reversed no. $= 10x + x + 3$ or $11x + 3$. $\therefore$ it is decreased by 27.

39. The difference in the time is 15 min. or $\frac{1}{4}$ hr. If x miles is the distance, $\frac{x}{27} - \frac{x}{28} = \frac{1}{4}$. $\therefore x = 189$.

40. See Ex. 27, page 160.

41. If the units is x, the tens is $2x$, and the hundreds $3x$, then $300x + 20x + x - 396 = 100x + 20x + 3x$; $x = 2$.

42. Let x be the greater and y the less, then $\frac{x}{y} = 5 + \frac{2}{y}$ or $x - 5y = 2$ and $\frac{12y}{x} = 2 + \frac{12}{x}$ or $2x - 12y = -12$; $x = 42$, $y = 8$.

43. $a + 2b + 2c + 2d = 46$, $2a + b + 2c + 2d = 43$, $2a + 2b + c + 2d = 41$, $2a + 2b + 2c + d = 38$. Add them and divide by 7, then $a + b + c + d = 24$ or $2a + 2b + 2c + 2d = 48$. Subtract each equation from this, and $a = 2$, $b = 5$, $c = 7$, $d = 10$.

3

44. Let x be the greater and y the less, then $x+y=ax$ and $x-y=by$. Then $a=\frac{x+y}{x}$, $b=\frac{x-y}{y}$. $\therefore a-b+ab=\frac{x+y}{x}-\frac{x-y}{y}+\frac{x^2-y^2}{xy}$ $=\frac{xy+y^2-x^2+xy+x^2-y^2}{xy}=2$.

Exercise 82 — Page 168

1. The straight line in Fig. 1 is the required graph. The point A shows that in 3 hr. he walks 12 mi., B shows that in 5 hr. he walks 20 mi.

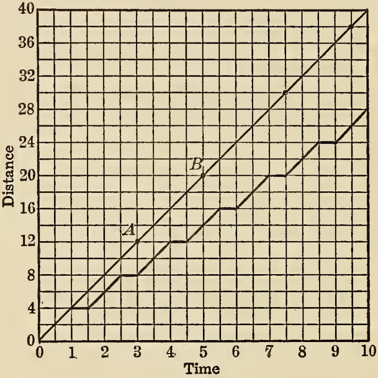


FIG. 1

2. The zigzag line in Fig. 1 is the required graph. It shows that he walks 8 miles in $2\frac{1}{2}$ hr., 12 in 4 hr., 14 in 5 hr., 7 in $2\frac{1}{4}$ hr., and 17 in $6\frac{1}{4}$ hr. Also in $1\frac{1}{2}$ hr. he goes 4 mi., in $3\frac{1}{2}$ hr. 10 mi., in $5\frac{1}{4}$ hr. 15 mi., and in $8\frac{1}{2}$ hr. 24 mi.

3. In Fig. 2, each unit on the vertical line represents 3 yards. The graphs show that at the end of 6 seconds A was at D and B at C and the distance between them was 6 spaces or 18 yards. A was 12 yards ahead of B when A was at F and B at E. This occurred 8 seconds after A started. The point G shows that B overtook A 12 seconds after A started.

4. In Fig. 3, the point A indicates that 12 oranges cost 19 cents. The point B shows that 7 would cost 11 cents, and C shows that for 5 cents

3 oranges could be bought. Similarly 10 would cost 16 cents, while for 3 cents 2 could be bought.

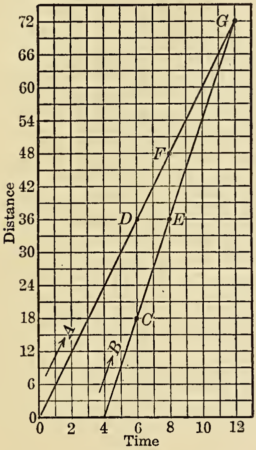


FIG. 2

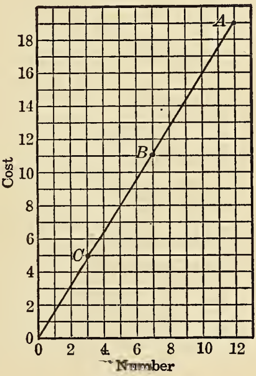


FIG. 3

5. The point A, in Fig. 4, shows that 8 k. equal 5 m. Similarly 2 m. = 3 k., 8 m. = 13 k., and so on. Also 3 k. = 2 m., 5 k. = 3 m., 11 k. = 7 m., 13 k. = 8 m., 16 k. = 10 m., 19 k. = 12 m., and 20 k. = 13 m., approximately.

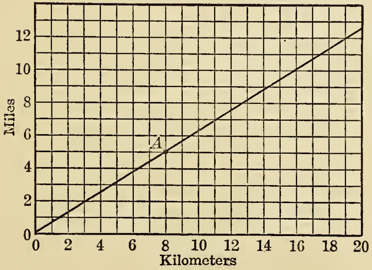


FIG. 4

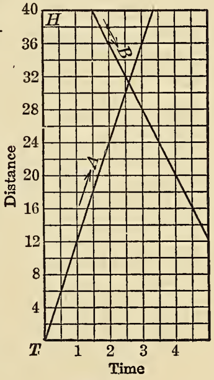


FIG. 5

6. The graphs are shown in Fig. 5. The point of intersection shows that they would meet about 2 hr. 35 m. or 2 hr. 40 m. after A started and that they would then be about 31 m. from Toronto.

7. In Fig. 6, each unit on the horizontal line represents 4 min. and on the vertical line 2 miles. At 10.30 the mail train is at y and the

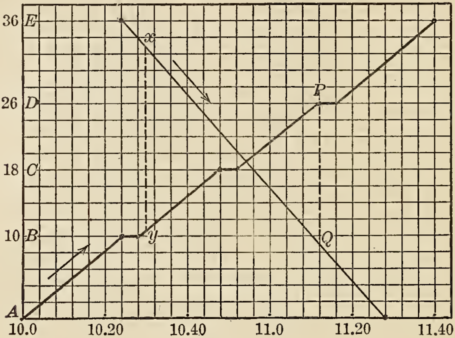


FIG. 6

express is at x, the distance between them being the length of the line xy, which is about 11 units or 22 miles. At 11.12 the distance is PQ, about $8\frac{1}{2}$ units or 17 miles. See Ex. 4 on page 167 of the text.

Exercise 83 — Page 170

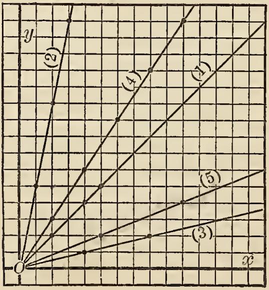


FIG. 7

1. In the diagram, Fig. 7, line (1) is the graph of $y = x$. It is constructed by finding a number of pairs of values of x and y. The points corresponding to these values are then joined as explained in Art. 122.

2–6. In Fig. 7, line (2) is the graph of $y = 5x$, (3) is the graph of $y = \frac{1}{4}x$ and $4y = x$, as these are equivalent equations, (4) is the graph of $y = \frac{3}{2}x$, and (5) is the graph of $5y = 2x$.

Exercise 84 — Page 172

1. The point (3, 4) is in the first quadrant since both coördinates are positive. (4, – 1) is in the fourth quadrant since x is positive and y negative. (– 5, 3) is in the second and (– 1, – 2) is in the third quadrant.

3. Both points lie on the axis of x and the distance between them is 8.

4. The point (0, 0) is on both axes and is therefore the origin. (0, 2) is on the y-axis and (– 5, 0), (4, 0) are on the x-axis.

5. See Art. 126. **9.** The figure is a rectangle whose sides are 6 and 9.

10. The base of the triangle lies on the x-axis and its length is 8. The length of the perpendicular from (3, 4) to the base is 4 and therefore the area is $\frac{1}{2} \times 4 \times 8 = 16$.

11. Draw the line joining (3, 2) and (7, 2), and the length of the perpendicular from (5, 8) will be seen to be 6 units.

Exercise 85 — Page 174

1. Two pairs are $x = 3$, $y = 3$. $x = 1$, $y = 5$. Plot these points and draw the line joining them.

2. It cuts each axis at a distance + 6 from the origin, that is, at the points (6, 0) and (0, 6).

3–8. The graphs of these six equations are shown in Fig. 8. To construct the graph of any one of them, find two pairs of values of x and y which satisfy the equation. Plot these two points and join them. Thus, the equation $y = 2x - 3$ is satisfied by $x = 0$, $y = -3$ and $x = 3$, $y = 3$. These two points are marked and joined, giving line (5) in the diagram.

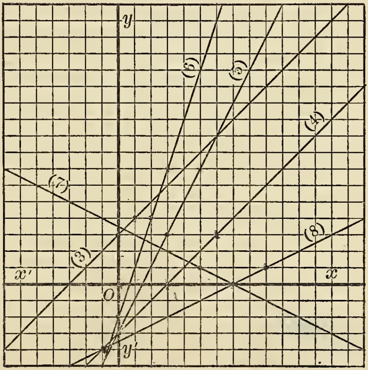


FIG. 8

12. The triangle is right angled and the area $= \frac{1}{2} \times 4 \times 6 = 12$.

13. When the graphs are constructed as in the preceding examples, they will be seen to intersect at the point $(9, -1)$, the coördinates of which satisfy both equations.

14. The point (3, 4) will lie on the graph if $x = 3$, $y = 4$ satisfies the equation which is true. The points (0, 8), (6, 0), (9, -4) also lie on it.

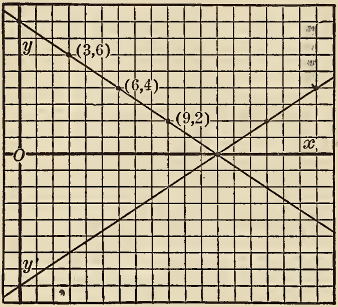


FIG. 9

15. The equation is satisfied by $x = 0$, $y = 8$ and $x = 12$, $y = 0$. These two points are joined, giving the graph shown in Fig. 9. The line is seen to pass through the points (3, 6), (6, 4), and (9, 2), and therefore $x = 3$, $y = 6$; $x = 6$, $y = 4$; $x = 9$, $y = 2$ are three pairs of values of x and y which satisfy the equation.

16. The graph of $2x - 3y = 24$ is also shown in Fig. 9. Since this line is extended indefinitely in the first quadrant, it is seen that there is an unlimited number of positive integral values of x and y which will satisfy its equation. Thus $x = 15$, $y = 2$; $x = 18$, $y = 4$; $x = 21$, $y = 6$, etc., satisfy the equation. But the graph of $2x + 3y = 24$ has only a limited part in the first quadrant and therefore the number of positive integral solutions is limited.

Exercise 86 — Page 176

1. The graph of $x = 3$ is a line parallel to the axis of y and at a distance 3 to the right of it. The graph of $x + 3 = 0$ is a line parallel to the axis of y and at a distance 3 to the left of it.

2. A line parallel to the x-axis at a distance 4 above it. A line parallel to the x-axis at a distance $-1\frac{1}{2}$ below it.

3. $x = 11 - 3y$, $y = \frac{1}{3}(11 - x)$. **4.** $x = \frac{2}{3}y + 2$, $y = \frac{3}{2}x - 3$.

5–10. The graphs of the equations in Examples 5–10 are shown in Figs. 10–15, respectively. In each case the line AB is the graph of the first equation and CD of the second equation, and P is the point of intersection of the graphs. The coördinates of P in Ex. 5 are (3, 2) and therefore $x = 3$, $y = 2$ is the solution. In Ex. 10 the coördinates of P are $(3, 1\frac{1}{2})$ and therefore $x = 3$, $y = 1\frac{1}{2}$ is the solution.

11. When the graphs are drawn, it will be found that the lines pass through a common point (3, 2).

12. See Art. 131. **13.** The first and second intersect at the point (5, − 1), the first and third at (3, 1) and the second and third at (4, − 3), but there is no point common to all three lines.

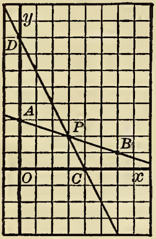


FIG. 10

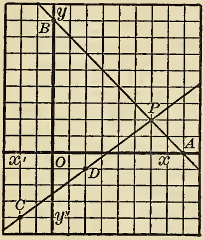


FIG. 11

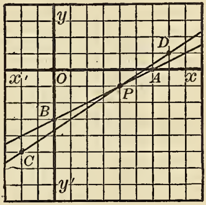


FIG. 12

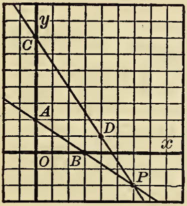


FIG. 13

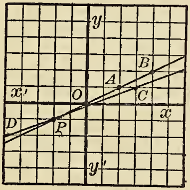


FIG. 14

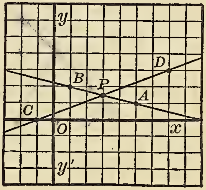


FIG. 15

14. The graph cuts the axis of x at the point $(-1\frac{1}{2}, 0)$ and the axis of y at the point (0, 3). It cuts the graph of $y = 6 - x$ at the point (1, 5).

15. When the graphs are drawn, the point of intersection of the first pair is the same as the point of intersection of the last pair. This point is (4, 3).

Exercise 87 — Page 177

1. It cuts the axis of x at (5, 0) and the axis of y at (0, 5). They intersect at the point (8, − 3).

2. Because their graphs, being straight lines, can intersect at only one point.

3. See Art. 126. **6.** See Ex. 5, page 169.

7. It is a right-angled triangle whose base is 5 and height 12.

8. See Ex. 14, page 175. **9.** The base is a horizontal line whose length is 15 and the height of the triangle is 15, therefore the area is

$$\tfrac{1}{2} \times 15 \times 15 = 112\tfrac{1}{2}.$$

10. The second triangle has the same base (8) and the same height (6) as the first triangle.

11–16. See Exs. 5–10, page 177.

17. It is a rectangle whose sides are 4 and 6.

19. The graphs all pass through a common point (2, 6) and therefore the three equations are satisfied by a common pair of roots $x = 2$, $y = 6$.

20. See Art. 131. **21.** The graphs of the three equations all pass through the point (6, 2) and therefore the equations are consistent.

22. The graphs of the first two equations intersect at the point (4, 3), but the graphs of the last two intersect at the point (8, 15). There is therefore no point common to the three and the equations are inconsistent.

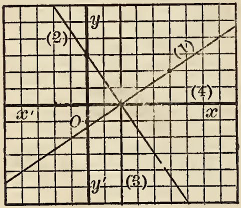


FIG. 16

23. The equations are consistent as in Ex. 21.

24. The coördinates of the vertices are (2, 4), $(-4, -1)$, $(6, -2)$.

27. In Fig. 16, line (1) is the graph of $2x - 3y = 1$, (2) of $3x + 2y = 8$, (3) of $x = 2$, (4) of $y = 1$. Since each pair of equations in the algebraic solution is represented by lines which pass through a common point (2, 1), it follows that the sets are equivalent.

Exercise 88 — Page 181

NOTE. — In this exercise only the factors of the expressions are given. The H. C. F. and the L. C. M. are given in the answers in the text.

2. $(x - y)(x + y)$, $y(x - y)$, $x(x - y)$.

3. $(a - b)(a + b)$, $b(a + b)$, $(a + b)^2$.

4. $(x - 3)(x - 4)$, $(x + 5)(x - 3)$, $(x - 3)(x + 3)$.

5. $(a + 3)(a + 5)$, $(a + 5)(a - 7)$, $(a + 5)(a - 2)$.

6. $3(x - 2)^2$, $3(x - 2)(x + 2)$, $3(x - 2)(x + 1)$.

7. $(x - y)(x + z)$, $y(x - y)$.

8. $(m - 2)(m^2 + 2m + 4)$, $m^2n^2(m - 2)(m + 2)$, $4(m - 2)^2$.

9. $6(a-b)(a^2+ab+b^2)$, $2a(a^2+ab+b^2)$.

10. $a(a+b-c)$, $(a+b-c)(a+b+c)$.

11. $(a-b-c)(a+b+c)$, $(b-c-a)(b+c+a)$, $(c-a-b)(c+a+b)$.

12. $(x+y)(x^2-xy+y^2)$, $(x^2+xy+y^2)(x^2-xy+y^2)$.

13. $(3x-2)(x+3)$, $(3x-2)(x-3)$, $(3x-2)(2x-3)$.

14. $(5x-1)(2a+3c)$, $(5x-1)(5x+1)$, $(5x-1)^2$.

15. $x(x-2)(x-3)$, $(x-3)(x^2+5)$.

16. $(u-v)(u+v)(u^2+v^2)$, $(u-v)(u^2+uv+v^2)$, $(u-v)(u+v)$, $u-v$. **17.** $(x+2)(x^2-8)$, $(x+3)(x^2-8)$.

18. $(x-3)(x-5)$, $(x-3)(x+4)$. H. C. F. $= x-3$. L. C. M. $= (x-3)(x+4)(x-5)$.

19. $a^2-5a+6=(a-3)(a-2)$. The L. C. M. $=(a-3)(a-2)(a+2)$. Then the other expression must be $(a-3)(a+2)=a^2-a-6$.

20. $x^3-7xy^2+6y^3=(x-2y)(x^2+2xy-3y^2)=(x-2y)(x-y)(x+3y)$. Each expression must have the factor $x-2y$ and to be trinomials one must have $x-y$ and the other $x+3y$.

Exercise 89 — Page 182

1. $(x-1)(x-2)$, $(x-1)(x^2-5x+3)$.

2. $(a-1)(a-5)$, $(a-1)(a^2-18a-1)$.

3. $(x-2)(x^2+4)$, $(x-2)(2x^2-3x-6)$.

4. $(a-1)(a^2+1)$, $(a-1)(3a^2+a+6)$.

5. $x(x-1)(x+4)$, $(x-1)(x-2)(x+3)$.

6. $(x-2)(x^2+5x+1)$, $(x-2)(x^2-2x-1)$.

7. $\dfrac{(a-2b)(a-b)}{(a-2b)(a-3b)(a+5b)}=\dfrac{a-b}{(a-3b)(a+5b)}$,
$\dfrac{x(x+1)(x-3)}{2x(x-3)(x+1)(x+2)}=\dfrac{1}{2(x+2)}$.

8. The L. C. M. must be divisible by the H. C. F. The L. C. M. $=(x^2-5x+6)(x-1)(x-4)$. Therefore one expression is $(x^2-5x+6)(x-1)$ and the other is $(x^2-5x+6)(x-4)$.

Exercise 90 — Page 186

1. Subtract (1) from (2) and we get $x^2-12x+35$ or $(x-5)(x-7)$. Evidently $x-7$ is not a common factor and it will be found that $x-5$ is and is therefore the only common factor.

2. On subtracting we get $8(a-3)(a-4)$. 8 is not a factor, but $a-3$ and $a-4$ are both common factors.

3. The difference $= 4(3x^2 + 2x + 2)$. On trial $3x^2 + 2x + 2$ is a common factor, so also is 2.

4. The diff. $= (2x - 9)(3x + 2)$. $2x - 9$ is the only common factor.

5. Multiply (1) by 2 and subtract and the result $= 7(2b^2 - b - 5)$.

6. The sum $= 9x^3 + 12x^2y - 77xy^2 = x(3x - 7y)(3x + 11y)$.

7. The diff. $= 3a(a^3 - 2a^2 + a - 2) = 3a(a - 2)(a^2 + 1)$. On trial $a - 2$ is the only common factor.

8. Multiply (2) by 3 and add and we get $x(2x^3 - 35x + 51)$. We must now find the H. C. F. of $2x^3 - 35x + 51$ and (2). Eliminate the x^3 and we get $(x - 3)(18x - 35)$, from which $x - 3$ is found to be the only common factor.

9. $3a^2b(6a^3 - a^2 - 4a - 1) = 3a^2b(a - 1)(3a + 1)(2a + 1)$.
$3a^2c(4a^3 - 2a^2 - 3a + 1) = 3a^2c(a - 1)(4a^2 + 2a - 1)$.

10. The factors of (1) are $(x - 1)(x^2 - 2)$, of which $x - 1$ is a factor of (2).

11. Subtract and we get $4x^2 - 11x - 3 = (x - 3)(4x + 1)$. Now $4x + 1$ cannot be a factor, but on trial $x - 3$ is a factor, then the expressions are $(x - 3)(x + 1)(x + 2)$, $(x - 3)(x^2 - x + 1)$.

12. The difference between (1) and (2) $= (x + 1)(x + 2)$. On trial $x + 1$ is found to be a factor of each. Then (1) $= (x + 1)(x + 2)(x + 3)$, (2) $= (x + 1)(x + 2)(x + 4)$, (3) $= (x + 1)(x + 3)(x + 4)$.

13. Multiply (1) by 3 and (2) by 2 and subtract and we get $17(x^2 + 3x - 1)$. Then (1) $= (x^2 + 3x - 1)(2x + 3)$, (2) $= (x^2 + 3x - 1)(3x - 4)$.

14. Diff. $= (x - 1)(x - 2)$. Then (1) $= (x - 1)(x - 2)(x - 3)$, (2) $= (x - 1)(x - 2)(x - 4)$.

15. $(4x^2 + 1)(5x^2 - 1)$, $(5x^2 - 1)^2$, $(5x^2 - 1)(5x^2 + x + 1)$.

16. Diff. $= 6(x - 3)(x + 1)$. On trial $x - 3$ is a common factor.

17. The other $= 70 \times 7 \div 14 = 35$. See Art. 135.

18. The other $= (x - 2)(x - 2)(x - 5)(x + 7) \div (x - 2)(x - 5)$.

19. Any common factor of two numbers is a factor of their difference, so that the only possible common factor is 11. (Art. 134.)

Exercise 91 — Page 187

1. $(x - 11)(x - 9)$, $(x - 11)(x - 13)$, $(x - 11)(x - 10)$.

2. $(x - 12)(x - 3)$, $(x - 3)(x^2 + 3x + 9)$, $(x - 3)(x^2 - 2)$.

3. $(a - b)(a + b)$, $(a - b)^2$, $(a - b)(a^2 + ab + b^2)$.

4. $x(x - 5)(x + 3)$, $(x + 3)(x + 2)(x - 4)$.

5. $(a-3)(2a-1)(2a+1)$, $(2a+1)(a+3)(a-3)$.

6. $(x-a)(x-b)$, $(x-b)(x-c)$.

7. $(x-1)(x-2)(x-3)$, $(x-1)(x+2)(x+3)$.

8. The diff. $=7x^3+22x^2-5x-24$. The sum $=x(2x^3-x^2-16x+15)$. Eliminate x^3 from $7x^3+22x^2-5x-24$ and $2x^3-x^2-16x+15$ and we get $51(x+3)(x-1)$. $x+3$ and $x-1$ are com. factors.

9. The diff. $=6a^3+41a^2+79a+30$. Eliminate the absolute terms and we get $a(10a^3+63a^2+113a+42)$. If we now eliminate a^3 from these, we get $8(a+3)(2a+1)$, and it will be found that $a+3$, $2a+1$ are com. factors.

10. The diff. $=8xy(x-y)^2$, and $(x-y)^2$ is a com. factor.

11. $(x^2+xy+y^2)(x^2-xy+y^2)$; $(x^2-xy+y^2)^2$.

12, 13. See Ex. 19 in the preceding exercise.

14. $(x^2+a^2)(x^4-a^2x^2+a^4)$, $(x^4+a^2x^2+a^4)(x^4-a^2x^2+a^4)$.

15. If the L. C. M. $=x$, then $dx=ab$. See Art. 135.

16. Let the quantities be ma, na, pa, the L. C. M. $=mnpa$. Product $=mnpa^3=mnpa\times a^2=a^2b$. The theorem is not true, however, if any two of m, n, p have a com. factor.

17. In the usual way $x-1$, $x-3$ are found to be com. factors.

18. See Ex. 20, page 181.

19. In the usual way $6x^2-5x+4$ is found to be a com. factor.

Exercise 92 — Page 189

1. $-\frac{1}{2}$. **2.** -2. **3.** $\frac{2}{3}$. **4.** 3. **5.** $-\frac{a}{y}$. **6.** $-\frac{5}{m}$.

7. $-2a$. **8.** $\frac{a}{b}$. **9.** $\frac{15}{7}$. **10.** a. **11.** $-\frac{ab}{c}$. **12.** $\frac{xy}{ab}$.

13. $\frac{a-b}{3}$. **14.** $-\frac{a-b}{b}$. **15.** $\frac{a-b}{y-x}$. **16.** $\frac{a-b}{3(d-c)}$.

17. $\frac{5}{c-d}$. **18.** $\frac{-xy}{c-d}$. **19.** $\frac{b-a}{c-d}$. **20.** $\frac{m(x-y)}{c-d}$. **21.** $\frac{4}{7}$.

22. $\frac{4}{7}$. **23.** $\frac{-a}{b}$. **24.** $\frac{b-a}{c}$. **25.** $\frac{x}{b-a}$. **26.** $\frac{a+b}{b-a}$.

27. $\frac{2-x}{x-y}$. **28.** $\frac{d-c}{d+c}$. **29.** They differ in sign; they differ in sign; they are equal.

30. $\frac{(p-q)(r-q)}{(x-y)(z-y)}=\frac{(q-p)(q-r)}{(y-x)(y-z)}=\frac{(q-p)(q-r)}{(x-y)(z-y)}=\frac{(p-q)(r-q)}{(y-x)(y-z)}$.

31. The first, third, and fifth are equal, so also are the second and fourth.

Exercise 93 — Page 191

1. $\dfrac{(a+2)(a+1)}{(a+2)(a+3)}$. **2.** $\dfrac{(x-y)(x+8y)}{(x-3y)(x+8y)}$. **3.** $\dfrac{(3x-1)(2x+1)}{(2x+3)(2x+1)}$.

4. $\dfrac{(x+y+z)(x+y-z)}{(x+y+z)(x-y-z)}$. **5.** $\dfrac{(a+b)(a^2+b^2)}{(a+b)^2}$.

6. $\dfrac{(2+y)(1-2y)}{(4+3y)(1-2y)}$. **7.** $\dfrac{(a+1)^2}{(a+1)(a^2+a+1)}$. **8.** $\dfrac{x(x-2)(x+1)}{(x+1)(x-2)^2}$.

9. $\dfrac{(a-1)(a-3)}{(a-3)(4a^2+3a-6)}$. **10.** $\dfrac{3(x-3)(x+2)}{6x^3(x-3)(x+1)}$.

11. $\dfrac{(x-1)(2x^2+x+3)}{(x+1)(2x^2+x+3)}$. **12.** $\dfrac{(3x+4)(x^2-2)}{(3x+4)(12x^2-7x-4)}$.

13. The factors of the numerator are $(a+1)(a^2-3)$ and $a+1$ will divide into the denominator.

14. First remove the com. factor $2x$. The H. C. F. of the numerator and denominator will then be found to be x^2+x+2, by the method of Art. 134.

Exercise 94 — Page 192

7. $\dfrac{2x^2}{(x-y)(x+y)}-\dfrac{2x}{x+y}=\dfrac{2x^2-2x(x-y)}{(x-y)(x+y)}$.

8. $\dfrac{1}{x+y}+\dfrac{1}{2x-y}=\dfrac{3x}{(x+y)(2x-y)}$.

9. $\dfrac{x-2y}{x}-\dfrac{x-2y}{x}=0$.

12. $\dfrac{(x-5)(x+2)}{(x-5)(x-3)}-\dfrac{(x+3)(x-1)}{(x-2)(x-1)}=\dfrac{x+2}{x-3}-\dfrac{x+3}{x-2}$.

13. $$\dfrac{x-y}{y}+\dfrac{2x}{x-y}-\dfrac{x^2(x+y)}{y(x+y)(x-y)}=\dfrac{x-y}{y}+\dfrac{2x}{x-y}-\dfrac{x^2}{y(x-y)}$$
$$=\dfrac{y^2}{y(x-y)}=\dfrac{y}{x-y}.$$

15. $$\dfrac{x+y}{x-y}+\dfrac{4xy}{x^2-y^2}+\dfrac{x-y}{x+y}=\dfrac{(x+y)^2+4xy+(x-y)^2}{x^2-y^2}$$
$$=\dfrac{2(x+y)^2}{x^2-y^2}=\dfrac{2(x+y)}{x-y}.$$

18. $\dfrac{2a+3b+2a+9b+2a-3b}{4a^2-9b^2}=\dfrac{3(2a+3b)}{4a^2-9b^2}=\dfrac{3}{2a-3b}$.

22. $$\dfrac{(a-b+c)(a+b-c)}{(b+c-a)(b-c+a)}-\dfrac{(c+a+b)(c+a-b)}{(b+c+a)(b+c-a)}$$
$$=\dfrac{a-b+c}{b+c-a}-\dfrac{c+a-b}{b+c-a}=0.$$

23. $\dfrac{a+b-c}{a+b+c}+\dfrac{b+c-a}{b+c+a}+\dfrac{c+a-b}{b+c+a}=\dfrac{a+b+c}{a+b+c}=1$.

24. $\dfrac{a}{a+1}-\dfrac{3a}{2(3a-2)}+\dfrac{5a}{2(a+1)(3a-2)}=\dfrac{3a^2-2a}{2(a+1)(3a-2)}$
$=\dfrac{a}{2(a+1)}.$

26. $\dfrac{1}{x-y}-\dfrac{1}{x+y}=\dfrac{2y}{x^2-y^2},\ \dfrac{2y}{x^2-y^2}-\dfrac{2y}{x^2+y^2}=\dfrac{4y^3}{x^4-y^4}.$
$\dfrac{4y^3}{x^4-y^4}-\dfrac{4y^3}{x^4+y^4}=\dfrac{4y^3(x^4+y^4)-4y^3(x^4-y^4)}{x^8-y^8}=\dfrac{8y^7}{x^8-y^8}.$

27. $\dfrac{1}{3-x}-\dfrac{1}{3+x}=\dfrac{2x}{9-x^2},\ \dfrac{2x}{9-x^2}-\dfrac{2x}{9+x^2}=\dfrac{4x^3}{81-x^4}.$

29. $\dfrac{1}{4(1-x)}-\dfrac{1}{4(1+x)}=\dfrac{2x}{4(1-x^2)}=\dfrac{x}{2(1-x^2)},\ \dfrac{x}{2(1-x^2)}+\dfrac{x}{2(1+x^2)}$
$=\dfrac{x(1+x^2)+x(1-x^2)}{2(1-x^4)}=\dfrac{x}{1-x^4},\ \dfrac{x}{1-x^4}-\dfrac{x}{1+x^4}=\dfrac{2x^5}{1-x^8}.$

30. $\dfrac{x}{(x+2)(x+3)}+\dfrac{15}{(x+2)(x+7)}-\dfrac{12}{(x+3)(x+7)}=\dfrac{1}{4}.$
$\therefore\ \dfrac{x(x+7)+15(x+3)-12(x+2)}{(x+2)(x+3)(x+7)}$ or $\dfrac{(x+3)(x+7)}{(x+2)(x+3)(x+7)}$ or $\dfrac{1}{x+2}=\dfrac{1}{4}.$

Exercise 95 — Page 195

1. $\dfrac{1}{a(a+x)}-\dfrac{1}{x(a+x)}=\dfrac{x-a}{ax(a+x)}.$

2. $\dfrac{6}{2a-3b}+\dfrac{3}{2a-3b}=\dfrac{9}{2a-3b}.$

3. $\dfrac{x+3}{x-2}+\dfrac{x-3}{x-2}=\dfrac{2x}{x-2}.$

4. $\dfrac{x}{x^2-9y^2}-\dfrac{1}{x-3y}=\dfrac{x-x-3y}{x^2-9y^2}.$

5. $\dfrac{2}{a(a-b)}+\dfrac{3}{b(a-b)}.$

6. $\dfrac{x+a}{x-a}+\dfrac{x^2-a^2}{a(x-a)}=\dfrac{ax+a^2+x^2-a^2}{a(x-a)}.$

7. $\dfrac{2}{x-1}-\dfrac{3}{x+1}+\dfrac{x^2-3}{x^2-1}.$

8. $\dfrac{5}{x-2}-\dfrac{4}{x+2}-\dfrac{16}{x^2-4}=\dfrac{x+2}{x^2-4}=\dfrac{1}{x-2}.$

10. $\dfrac{3}{x+4}-\dfrac{3}{x-4}+\dfrac{24}{x^2-16}=\dfrac{3(x-4)-3(x+4)+24}{x^2-16}=\dfrac{0}{x^2-16}=0.$

11. $\dfrac{1}{b+3a}+\dfrac{1}{b-3a}+\dfrac{6a}{b^2-9a^2}=\dfrac{2b+6a}{b^2-9a^2}=\dfrac{2}{b-3a}.$

12. $$\frac{3a+2x}{3a-2x}-\frac{3a-2x}{3a+2x}-\frac{16x^2}{9a^2-4x^2}=\frac{(3a+2x)^2-(3a-2x)^2-16x^2}{9a^2-4x^2}$$
$$=\frac{8x(3a-2x)}{9a^2-4x^2}=\frac{8x}{3a+2x}.$$

13. $$\frac{1}{x-1}+\frac{4}{x-1}-\frac{8}{x+1}+\frac{3x+7}{x^2-1}.$$

14. $$\frac{a}{c(a-b)}+\frac{c}{a(a-b)}.$$

15. $$\frac{1}{(a-b)(c-a)}-\frac{1}{(a-b)(c-b)}=\frac{a-b}{(a-b)(c-a)(c-b)}.$$

16. $$\frac{a}{(x-a)(a-b)}-\frac{b}{(x-b)(a-b)}=\frac{ax-bx}{(x-a)(x-b)(a-b)}$$
$$=\frac{x}{(x-a)(x-b)}.$$

17. $$\frac{-2}{(x-y)(x+y)}+\frac{2}{(x-y)^2}+\frac{1}{x(x+y)}$$
$$=\frac{-2x(x-y)+2x(x+y)+(x-y)^2}{x(x+y)(x-y)^2}$$
$$=\frac{(x+y)^2}{x(x+y)(x-y)^2}=\frac{x+y}{x(x-y)^2}.$$

19. $$\frac{2}{(x-5)(x-3)}+\frac{2}{(x-3)(x-1)}-\frac{4}{(x-5)(x-1)}$$
$$=\frac{2(x-1)+2(x-5)-4(x-3)}{(x-1)(x-3)(x-5)}=0.$$

20. $$\frac{(a+b)(a-b)+(b+c)(b-c)+(c+a)(c-a)}{(a-b)(b-c)(c-a)}=0.$$

21. $$\frac{-1}{(a-b)(c-a)}-\frac{1}{(b-c)(a-b)}-\frac{1}{(c-a)(b-c)}$$
$$=\frac{-b+c-c+a-a+b}{(a-b)(b-c)(c-a)}=0.$$

22. $$\frac{-a^2(b-c)-b^2(c-a)-c^2(a-b)}{(a-b)(b-c)(c-a)}=\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=1.$$

24. $$\frac{bc-ax}{(a-b)(c-a)}+\cdots$$
$$=\frac{(bc-ax)(b-c)+(ca-bx)(c-a)+(ab-cx)(a-b)}{(a-b)(b-c)(c-a)}$$
$$=\frac{b^2c-bc^2+c^2a-ca^2+a^2b-ab^2}{(a-b)(b-c)(c-a)}=-1.$$

25. $$\frac{-a^2}{(a^2-b^2)(c^2-a^2)}-\frac{b^2}{(b^2-c^2)(a^2-b^2)}-\frac{c^2}{(c^2-a^2)(b^2-c^2)}=0.$$

26. $\dfrac{-\ bc(a+d)}{(a-b)(c-a)}-\dfrac{ca(b+d)}{(b-c)(a-b)}-\dfrac{ab(c+d)}{(c-a)(b-c)}$

$$=\frac{-\ bc(b-c)(a+d)-ca(c-a)(b+d)-ab(a-b)(c+d)}{(a-b)(b-c)(c-a)}$$

$$=\frac{d(-b^2c+bc^2-c^2a+ca^2-a^2b+ab^2)}{(a-b)(b-c)(c-a)}=d.$$

27. $\dfrac{1}{a-b}-\dfrac{1}{2(a+b)}=\dfrac{a+3\,b}{2(a^2-b^2)},$

$\dfrac{a+3\,b}{2(a^2-b^2)}-\dfrac{a+3\,b}{2(a^2+b^2)}=\dfrac{2\,ab^2+6\,b^3}{2(a^4-b^4)}=\dfrac{ab^2+3\,b^3}{a^4-b^4},$

$\dfrac{ab^2+3\,b^3}{a^4-b^4}-\dfrac{4\,b^3}{a^4-b^4}=\dfrac{b^2(a-b)}{a^4-b^4}.$

28. $\dfrac{2\,x}{x^2-25}-\dfrac{2\,x}{x^2-9}=\dfrac{2\,x(x^2-9)-2\,x(x^2-25)}{(x^2-25)(x^2-9)}=\dfrac{32\,x}{(x^2-25)(x^2-9)}.$

29. $\dfrac{1}{a+4}-\dfrac{1}{a+1}=\dfrac{-3}{(a+4)(a+1)},\quad \dfrac{3}{a+2}-\dfrac{3}{a+3}=\dfrac{3}{(a+2)(a+3)}.$

$\dfrac{-3}{(a+4)(a+1)}+\dfrac{3}{(a+2)(a+3)}$

$$=\frac{-3(a^2+5\,a+6)+3(a^2+5\,a+4)}{(a+1)(a+2)(a+3)(a+4)}.$$

30. $\dfrac{2\,x}{x^2-9}-\dfrac{2\,x}{x^2-1}=\dfrac{2\,x(x^2-1)-2\,x(x^2-9)}{(x^2-9)(x^2-1)}.$

Exercise 96 — Page 198

7. $\left(x^2+\dfrac{1}{x^2}\right)^2-1=x^4+2+\dfrac{1}{x^4}-1.$

8. $\left(a^2+\dfrac{2}{a^2}\right)^2-4=a^4+\dfrac{4}{a^4}.$

9. $\dfrac{b(a+x)}{a}\times\dfrac{x}{a+x}\times\dfrac{a}{bx}=1.$

11. $\dfrac{(x-5)(x-6)}{(x-3)^2}\times\dfrac{x(x-3)}{x(x-5)}=\dfrac{x-6}{x-3}.$

14. $\left(\dfrac{x}{y}-\dfrac{y}{x}\right)\left(\dfrac{x}{y}+\dfrac{y}{x}\right)+\dfrac{x}{y}-\dfrac{y}{x}\div\left(\dfrac{x}{y}-\dfrac{y}{x}\right)=\dfrac{x}{y}+\dfrac{y}{x}+1.$

17. $\dfrac{a+1}{a(a-2)}\times\dfrac{a+2}{a(a-1)}\times\dfrac{a^2(a-1)(a+1)}{(a-2)(a+2)}.$

18. $\dfrac{(a-2)(a+2)}{a(a+5)}\times\dfrac{(a+5)(a-5)}{a(a+2)}.$

19. $\dfrac{(2\,x-1)(x+1)}{(x-1)(x-3)}\times\dfrac{(2\,x-3)(x-1)}{(3\,x+2)(2\,x-1)}\times\dfrac{(x-3)(3\,x+2)}{(x-2)(2\,x-3)}=\dfrac{x+1}{x-2}.$

20. $\dfrac{a(a+x)-x(a-x)+2ax}{a^2-x^2}\div(a+x)^2=\dfrac{(a+x)^2}{a^2-x^2}\div(a+x)^2.$

21. $\dfrac{a+b}{a}\times\dfrac{a+c}{a}\times\dfrac{b}{b+a}\times\dfrac{c}{c+a}=\dfrac{bc}{a^2}.$

23. $\dfrac{(a-8)(a+8)}{(a+8)(a+16)}\times\dfrac{(a-4)(a+16)}{(a-4)(a^2+4a+16)}\times\dfrac{a^2+4a+16}{(a-8)^2}=\dfrac{1}{a-8}.$

24. $\dfrac{5a}{a-6b}-\dfrac{2b}{3a-2b}=\dfrac{3(5a^2-4ab+4b^2)}{(a-6b)(3a-2b)},\quad \dfrac{2a}{a+2b}-\dfrac{a-2b}{3a-2b}$

$=\dfrac{5a^2-4ab+4b^2}{(a+2b)(3a-2b)}.\quad$ Quotient $=\dfrac{3(a+2b)}{(a-6b)}.$

Exercise 97 — Page 200

8. Numerator $=\dfrac{6}{a-3}$. Denominator $=\dfrac{a^2}{a-3}$, fraction $=\dfrac{6}{a^2}.$

10. Num. $=\dfrac{x^2y^2+2xy+1}{xy}=\dfrac{(xy+1)^2}{xy}.$ Den. $=(xy+1)(xy-1).$

11. Num. $=\dfrac{b^3}{a^2-ab+b^2}.$ Den. $=\dfrac{a^3}{a^2-ab+b^2}.$

12. Num. $=\dfrac{bc+c^2-a^2-ab}{(a+b)(b+c)}=\dfrac{(c-a)(a+b+c)}{(a+b)(b+c)}.$

Den. $=\dfrac{(a-b)(a+b+c)}{(b+c)(c+a)}.$

13. Den. $=2x+3-\dfrac{3(x+6)}{6}=\dfrac{3x}{2}.$ Fraction $=3\times\dfrac{2}{3x}=\dfrac{2}{x}.$

14. $\dfrac{a^2(a+b)}{a^2-ab}+\dfrac{b^2(a+b)}{b^2-ab}=\dfrac{a(a+b)}{a-b}-\dfrac{b(a+b)}{a-b}=\dfrac{a^2-b^2}{a-b}=a+b.$

15. Num. $=\dfrac{a^3+b^3}{a^2b^2}=\dfrac{(a+b)(a^2-ab+b^2)}{a^2b^2}.$ Den. $=\dfrac{b^2-ab+a^2}{a^2b^2}.$

16. $x-y=\dfrac{4ab}{a^2-b^2},\ x+y=\dfrac{2(a^2+b^2)}{a^2-b^2}.$

17. $1-2a+b=1-\dfrac{2(x-y)}{x+y}+\dfrac{(x-y)^2}{(x+y)^2}=\dfrac{(x+y)^2-2(x^2-y^2)+(x-y)^2}{(x+y)^2}$

$=\dfrac{4y^2}{(x+y)^2},\ 1+2a+b=\dfrac{4x^2}{(x+y)^2}.$ Then $\dfrac{1-2a+b}{1+2a+b}=\dfrac{y^2}{x^2}.$

Exercise 98 — Page 201

1. $\dfrac{a}{a+b}+\dfrac{a}{a-b}=\dfrac{2a^2}{a^2-b^2},\quad\dfrac{2a^2}{a^2-b^2}+\dfrac{2a^2}{a^2+b^2}=\dfrac{4a^4}{a^4-b^4}.$

2. $\dfrac{ac}{b}-\dfrac{a}{bc}=\dfrac{ac^2-a}{bc}.$

3. $\dfrac{x}{x+3y}+\dfrac{y}{x-3y}-\dfrac{(x-y)^2}{x^2-9y^2}.$

4. $\dfrac{2\,xy}{y-x}+\dfrac{xy}{x-y}-\dfrac{xy}{y-x}=\dfrac{2\,xy}{y-x}-\dfrac{xy}{y-x}-\dfrac{xy}{y-x}=0.$

6. $\dfrac{x^2-(x^2-x)-1+x-1}{x(x-1)}=\dfrac{2\,x-2}{x(x-1)}=\dfrac{2}{x}.$

8. $\dfrac{1-3\,x+3\,x^2-x^3+1+3\,x+3\,x^2+x^3}{1+2\,x+x^2-1+2\,x-x^2}=\dfrac{2+6\,x^2}{4\,x}=\dfrac{1+3\,x^2}{2\,x}.$

9. $\dfrac{(b-c)^2}{b^2+c^2}\times\dfrac{b^2+c^2}{b-c}\times\dfrac{b^2-bc+c^2}{b^2}\times\dfrac{b^2}{(b+c)(b^2-bc+c^2)}.$

10. $\dfrac{1}{x+1}-\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+1)(x+2)(x+3)}=\dfrac{(x+2)^2}{(x+1)(x+2)(x+3)}.$

11. $\dfrac{a}{a^2+ab+b^2}+\dfrac{b}{a^2-ab+b^2}+\dfrac{2\,a^2b}{(a^2+ab+b^2)(a^2-ab+b^2)}$

$=\dfrac{a^3+2\,a^2b+2\,ab^2+b^3}{(a^2+ab+b^2)(a^2-ab+b^2)}=\dfrac{(a+b)(a^2+ab+b^2)}{(a^2+ab+b^2)(a^2-ab+b^2)}.$

12. $\dfrac{(a^2-b^2)(a^4+a^2b^2+b^4)}{(a+b)^2}\times\dfrac{a^2+b^2}{(a+b)(a-b)}\times\dfrac{a+b}{a^4+a^2b^2+b^4}.$

14. $(1-a^2)\div\left\{(1-a)^2-(a^3-1)\times\dfrac{a+1}{a^2+a+1}\right\}$

$=(1-a^2)\div(1-2\,a+a^2-a^2+1)=(1-a^2)\div 2(1-a)=\dfrac{1+a}{2}.$

15. $\dfrac{-(b-c)(b+c-a)-(c-a)(c+a-b)-(a-b)(a+b-c)}{(a-b)(b-c)(c-a)}=0.$

17. $\dfrac{2(2\,x+3)}{(2\,x+3)(3\,x-2)}+\dfrac{3(x-3)}{(x-3)(2\,x+3)}-\dfrac{5(3\,x-2)}{(3\,x-2)^2}$

$=\dfrac{2}{3\,x-2}+\dfrac{3}{2\,x+3}-\dfrac{5}{3\,x-2}.$

18. $\dfrac{(x-4)(x-2)}{x+1}\times\left(\dfrac{-1}{(x-1)(x-2)}+\dfrac{1}{(x-2)(x-4)}+\dfrac{1}{(x-4)(x-1)}\right)$

$=\dfrac{-(x-4)}{(x-1)(x+1)}+\dfrac{1}{x+1}+\dfrac{x-2}{(x-1)(x+1)}=\dfrac{x+1}{x^2-1}=\dfrac{1}{x-1}.$

19. $\dfrac{x}{x+a}+\dfrac{a}{x+a}=\dfrac{x+a}{x+a}=1,\ \dfrac{x}{x+b}+\dfrac{b}{x+b}=1,\ \dfrac{x}{x+c}+\dfrac{c}{x+c}=1.$

20. $\dfrac{2\,x^2-xy}{x^2-y^2}\div\dfrac{2\,x-y}{x^2-y^2}.$

22. $1-\dfrac{1}{a}+1-\dfrac{1}{b}+1-\dfrac{1}{c}-1=2-\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right).$

23. It is evident when the terms of the fractions are simplified.

24. $a=\dfrac{2}{2-b}=\dfrac{2}{2-\dfrac{2}{2-c}}=\dfrac{2(2-c)}{2-2\,c}=\dfrac{2-c}{1-c}=\dfrac{2-\dfrac{2}{2-d}}{1-\dfrac{2}{2-d}}=\dfrac{2-2\,d}{-d}$

$=2-\dfrac{2}{d}=2-(2-x)=x.$

25. $\dfrac{1+x^3}{1+x^2}\times\left(1+x+\dfrac{x-1}{x^2-x+1}\right)=\dfrac{(1+x)(1-x+x^2)}{1+x^2}\times\dfrac{x(1+x^2)}{x^2-x+1}.$

26. $\dfrac{a}{b}-\dfrac{a+10}{b+10}=\dfrac{10(a-b)}{b(b+10)}.$ Since $a>b$ the remainder is positive.

27.
$$\frac{(1+ab)(1+ac)(c-b)+(1+bc)(1+ba)(a-c)+(1+ca)(1+cb)(b-a)}{(a-b)(b-c)(c-a)}.$$

When the numerator is multiplied out and simplified it becomes $a^2b-ab^2+b^2c-bc^2+c^2a-ca^2$ which differs only in sign from the denominator.

28. When $x=a+b$, $\dfrac{1}{x-a}+\dfrac{1}{x-b}=\dfrac{1}{b}+\dfrac{1}{a}=\dfrac{a+b}{ab}.$

When $x=\dfrac{2\,ab}{a+b}$, $\dfrac{1}{x-a}+\dfrac{1}{x-b}=\dfrac{a+b}{a(b-a)}+\dfrac{a+b}{b(a-b)}$

$$=\frac{a+b}{a(b-a)}-\frac{a+b}{b(b-a)}=\frac{b^2-a^2}{ab(b-a)}=\frac{b+a}{ab}.$$

29. To prove that $xy=(x+y)\div\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$ or $(x+y)\div\dfrac{x+y}{xy}.$

30. Find the product of x, y, z and also their sum and the required result follows.

31. $1-a=\dfrac{2\,y}{x+y}$, $1-b=\dfrac{2\,z}{y+z}$, $1-c=\dfrac{2\,x}{z+x}$, $1+a=\dfrac{2\,x}{x+y}$, etc.

32. $\dfrac{a+2\,b}{a+b}-\dfrac{a+3\,b}{a+2\,b}=\dfrac{b^2}{(a+b)(a+2\,b)}$, which is positive.

33. See Ex. 27, page 196.

34. $\dfrac{x-y}{(z-x+y)(z+x-y)}+\dfrac{y-z}{(x+y-z)(x-y+z)}+\dfrac{z-x}{(y-z+x)(y+z-x)}$

$$=\frac{(x-y)(x+y-z)+(y-z)(y+z-x)+(z-x)(z+x-y)}{(x-y+z)(y-z+x)(z-x+y)}=0.$$

35. $\dfrac{x}{(x-4)(x-3)}-\dfrac{x}{(x-1)(x-2)}=\dfrac{x(x^2-3\,x+2)-x(x^2-7\,x+12)}{(x-1)(x-2)(x-3)(x-4)}.$

36. $\dfrac{(x-2\,a)(x-2\,c)-(x+2\,a)(x+2\,c)+4\,ac}{x^2-4\,c^2}=\dfrac{-4(ax+cx-ac)}{x^2-4\,c^2}$

$=0$, since $ax+cx=ac$.

37. $\dfrac{x(1+xy)+y-x}{1+xy-x(y-x)}-\dfrac{y(1-xy)-y+x}{1-xy-y(y-x)}=\dfrac{y(1+x^2)}{1+x^2}-\dfrac{x(1-y^2)}{1-y^2}=y-x.$

Then given expression $=(y-x)\times\dfrac{xy}{y^2-x^2}=\dfrac{xy}{y+x}.$

Exercise 99 — Page 205

1. $3(3x+1)=2(x+12)$ or $9x+3=2x+24$ or $x=3$.

2. $(3x-5)(5x-1)=(5x-3)(3x-1)$ or $15x^2-28x+5$
$=15x^2-14x+3$ or $x=\frac{1}{7}$.

3. $11x-3(x-1)=33(x-9)$ or $8x+3=33x-297$ or $x=12$.

4. $(x-2)(x-5)=(x-3)(x-6)$ or $x=4$.

5. $14(x+2)=35(x-2)-10(x-1)$ or $x=8$.

6. $39(x+1)-26(x-3)=6(x+30)$ or $x=9$.

7. $3(7x-3)+6=2(2x+5)$ or $x=\frac{13}{17}$.

8. $x^3+6x^2-13x+6=x^3+6x^2-5x-10$ or $x=2$.

9. $(x+3)(x-3)=2(x-1)(x-3)-(x+1)(x-1)$ or $x=2$.

10. $x(x+1)+(x-1)(x+1)=2x(x-1)$ or $x=\frac{1}{3}$.

11. $(x-2)(x-3)+2(x+2)(x-3)=3(x+2)(x-2)$ or $x=\frac{6}{7}$.

12. $6x^2-31x+5=6x^2+5x-6$ or $x=\frac{11}{36}$.

13. $(2x+38)(2x+1)=(6x+8)(x+12)-(x+12)(2x+1)$ or $x=2$.

14. $y^3-20y^2+126y-240=y^3-20y^2+111y-180$ or $y=4$.

15. $\dfrac{8x+28-8x-19}{12}$ or $\dfrac{3}{4}=\dfrac{5x+11}{7x+9}$ or $x=17$.

16. $(x-1)(x-3)+(x-2)(x-5)=2(x-2)(x-3)$ or $x=1$.

17. $(6-8x)(1-x)+3(3-x)=8(3-x)(1-x)$ or $x=\frac{3}{5}$.

18. The first side $=\dfrac{5x^2+9x+9}{x^2+3x+2}$. Cross multiply and $x=9$.

19. Cross multiply and $x=2$.

20. $(2x-2)(x-5)=(x-3)(x-4)+(x-3)(x-5)$ or $x=5\frac{2}{3}$.

21. $\dfrac{4-6x}{3}+4x-\dfrac{5x-10}{9}=\dfrac{2x-3}{9}+\dfrac{61}{18}$ or $x=\frac{1}{2}$.

22. $x+\dfrac{5}{3x-4}=x+\dfrac{7}{4x-5}$ or $x=3$.

23. $\frac{1}{2}x+\frac{1}{5}(x+1)+\frac{1}{6}(x+2)=30$ or $x=34$.

24. If the parts are x and $300-x$ then $\dfrac{x}{5}-\dfrac{300-x}{7}=18$ from which $x=177\frac{1}{2}$ or $\dfrac{300-x}{7}-\dfrac{x}{5}=18$ and $x=72\frac{1}{2}$.

25. If x gal. are added then 4 % of $(100+x)=4$ or $x=33\frac{1}{3}$.

26. If it was x then $5(x-3)=\frac{1}{5}(x+3)$ or $x=3\frac{1}{4}$. 3

27. If the tea cost $15x$ c. per pound then the coffee cost $8x$. The gain on the coffee $=4x\times 560$ and the loss on the tea $=3\frac{3}{4}x\times 180$. $\therefore 2240x-675x=6260$ or $x=4$ and the tea cost 60 c.

28. If I sold x lb. the selling price was $25x$ cents, then $25x + 5 = 27(x - 1)$ or $x = 16$.

29. If the distance is x yd. then $\frac{6}{10}x - \frac{5}{11}x = 600$ or $x = 4125$.

Exercise 100 — Page 208

1. Simplify the first side and cross multiply and $x = 2$.

2. Rearrange as $\frac{2x+3}{4} - \frac{x+2}{2} = \frac{x-1}{6x-8}$. Proceed as in Ex. 1, $x = 1\frac{1}{5}$.

3. $\frac{x+3}{7} - \frac{2x+1}{14} = \frac{3x+5}{6x+2}$ or $\frac{5}{14} = \frac{3x+5}{6x+2}$ or $x = -5$.

4. $\frac{6x+1}{4} - \frac{3x-1}{2} = \frac{2x-1}{3x-2}$ or $\frac{3}{4} = \frac{2x-1}{3x-2}$ or $x = 2$.

5. $\frac{4}{x-8} + \frac{3}{2(x-8)} - \frac{2}{3(x-8)} = \frac{29}{24}$ or $\frac{29}{6(x-8)} = \frac{29}{24}$ or $x = 12$.

6. $\frac{5}{12} = \frac{6x+7}{3(3x+2)} - \frac{5x-5}{4(3x+2)}$ or $\frac{5}{12} = \frac{9x+43}{12(3x+2)}$ or $x = 5\frac{1}{2}$.

7. $\frac{25}{18} = \frac{23x-88}{17x-66}$ or $x = 6$.

8. $\frac{3x-4}{3(2x-3)} + \frac{6x-5}{4(2x-3)} = \frac{1}{12}$ or $\frac{30x-31}{12(2x-3)} = \frac{1}{12}$ or $x = 1$.

9. $\frac{5x-17}{13-4x} = 1$ or $x = 3\frac{1}{3}$.

10. $\frac{1}{10} = \frac{4x-3}{2(2x-1)} + \frac{5x-7}{5(2x-1)}$ or $\frac{1}{10} = \frac{30x-29}{10(2x-1)}$ or $x = 1$.

11. $\frac{1}{x^2-3x+2} = \frac{1}{x^2-7x+12}$ or $x = 2\frac{1}{2}$.

12. $\frac{5}{x^2-15x+50} = \frac{5}{x^2-9x+14}$ or $x = 6$.

13. $\frac{1}{3(x+4)} + \frac{1}{6(x+4)} = \frac{3}{2(x+5)} - \frac{1}{x+6}$ or

$$\frac{1}{2(x+4)} = \frac{x+8}{2(x^2+11x+30)} \text{ or } x = -2.$$

14. $\frac{3}{x-5} - \frac{1}{x-5} = \frac{5}{x-7} - \frac{8}{x-7}$ or $\frac{2}{x-5} = \frac{-3}{x-7}$ or $x = 5\frac{4}{5}$.

15. $\frac{x-8}{x-10} - \frac{x-7}{x-9} = \frac{x-5}{x-7} - \frac{x-4}{x-6}$ or

$$\frac{2}{x^2-19x+90} = \frac{2}{x^2-13x+42} \text{ or } x = 8.$$

16. $\frac{2x-27}{x-14} - \frac{2x-17}{x-9} = \frac{x-12}{x-13} - \frac{x-7}{x-8}$ or $x = 11$.

18. $4 - \dfrac{1}{x-4} + 5 + \dfrac{2}{2x-3} = 4 - \dfrac{2}{2x-7} + 5 + \dfrac{1}{x-1}.$

$\therefore -\dfrac{1}{x-4} + \dfrac{2}{2x-3} = -\dfrac{2}{2x-7} + \dfrac{1}{x-1}$ or $x = 2\frac{1}{2}$.

19. $5 + \dfrac{1}{x-13} - 2 - \dfrac{1}{x-6} = 4 + \dfrac{1}{x-14} - 1 - \dfrac{1}{x-7}$ or $x = 10$.

20. $1 - \dfrac{2}{x+1} + 1 + \dfrac{3}{x-2} + 1 - \dfrac{1}{x-1} = 3.$

$\therefore \dfrac{3}{x-2} = \dfrac{2}{x+1} + \dfrac{1}{x-1}$ or $\dfrac{3}{x-2} = \dfrac{3x-1}{x^2-1}$ or $x = \frac{5}{7}$.

21. $\dfrac{x-a}{b+c} - 1 + \dfrac{x-b}{c+a} - 1 + \dfrac{x-c}{a+b} - 1 = 0$ or $\dfrac{x-a-b-c}{b+c} + \cdots = 0.$

$\therefore (x-a-b-c)\left(\dfrac{1}{b+c} + \dfrac{1}{c+a} + \dfrac{1}{a+b}\right) = 0.\quad \therefore x-a-b-c=0$ or $x = a+b+c$.

22. If $(a+b)(a-b) = a-b$ then either $a-b=0$ or $a+b=1$.

Exercise 101 — Page 210

1. $mx = b-a$ or $x = \dfrac{b-a}{m}$. **2.** $x(a-b) = 2$ or $x = \dfrac{2}{a-b}$.

3. $ab + x = bc$ or $x = bc - ab$. **4.** $3x + 3a = 7x - 7a$ or $x = \frac{5}{2}a$.

5. $bx - ab = ax + ab$ or $x = \dfrac{2ab}{b-a}$. **6.** $x = \dfrac{ac}{b}$. **7.** $x = \dfrac{a^2-ab}{a-b} = a$.

8. $x = \dfrac{b^2-a^2}{2(b-a)} = \dfrac{b+a}{2}$. **9.** $x = \dfrac{a^2+b^2+2ab}{a+b} = a+b$.

10. $\dfrac{ax}{b} + \dfrac{dx}{c} = \dfrac{b}{a} + \dfrac{c}{d}$ or $\dfrac{x(ac+bd)}{bc} = \dfrac{ac+bd}{ad}$ or $x = \dfrac{bc}{ad}$.

11. $x = \dfrac{a^2+b^2}{a+b}$. **12.** $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{2}{x}$ or $x = \dfrac{2ab}{a+b}$.

14. $x = \dfrac{ab-a^2}{a^2-b^2} = \dfrac{-a}{a+b}$. **15.** $x = \dfrac{a^3-b^3}{a^2+ab+b^2} = a-b$.

16. $\dfrac{ax-bx}{(x-a)(x-b)} = \dfrac{a-b}{x-c}$. Divide by $a-b$ and cross multiply.

19. $\dfrac{-a}{(x-a)(x-2a)} = \dfrac{-a}{(x-3a)(x-4a)}$ or $x^2 - 3ax + 2a^2$
$= x^2 - 7ax + 12a^2$.

20. $-2ax - 2bx = (a+b)^2$ or $x = \dfrac{(a+b)^2}{-2(a+b)} = -\dfrac{a+b}{2}$.

21. $b(a+x)(b+x) - ab(b+c) = a^2c + bx^2$, $x = \dfrac{a^2c+abc}{ab+b^2} = \dfrac{ac}{b}$.

23. $x = \dfrac{a^3-b^3}{a^2+b^2+ab} = a-b$.

24. If x is the number, then $x - a = 3(x - b)$.

25. Let the parts be bx and cx, then $bx + cx = a$.

26. Let the parts be x and $a - x$, then $mx - n(a - x) = b$.

27. If the side is x, then $x^2 = (x + a)(x - b)$.

28. If the number is x, then $a + \frac{x}{a} + \frac{1}{3}x = b$.

29. If there were x acres, then $\frac{x}{m} + a + \frac{x}{n} + b = x$.

31. $2s = an + ln$, $\therefore n = \frac{2s}{a + l}$, $a = \frac{2s - ln}{n}$, $l = \frac{2s - an}{n}$.

Exercise 102 — Page 213

1. Adding $2mx = a + b$ or $x = \frac{a + b}{2m}$, $y = \frac{a - b}{2n}$.

2. Multiply (1) by l and (2) by m and subtract, then $x = 0$, $y = 1$.

3. Multiply (2) by q and subtract, then $x = \frac{r}{p - q}$, $y = \frac{r}{q - p}$.

4. Multiply (2) by b and subtract, then $x = a$, $y = b$.

5. Multiply (1) by a and (2) by b and add, then $x = b$, $y = a$.

6. Multiply (1) by b and add, then $x = \frac{1}{a}$, $y = \frac{1}{b}$.

7. Multiply by a and b and add, $x = \frac{2a^3 + 2b^2a}{a^2 + b^2} = 2a$, $y = -3b$.

8. Multiply by $2a$ and b and add, $x = \frac{3b}{2}$, $y = -\frac{a}{2}$.

9. Multiply (2) by b and add, then $x = \frac{a}{a + b}$, $y = \frac{b}{a + b}$.

10. Adding $\frac{2x}{a} = 4$, $x = 2a$, $y = b$.

11. Remove fractions, multiply by b and a and add, $x = a$, $y = b$.

12. Remove fractions, multiply by a and b^2 and subtract, $x = a$, $y = -b$.

15. Multiply (1) by 4 and add, then $\frac{7a}{x} = 3\frac{1}{2}$ or $x = 2a$, $y = -b$.

16 Multiply (2) by $a - b$ and subtract, then $2bx = 2ab$ or $x = a$, $y = b$.

17 Multiply (1) by b_2 and (2) by b_1, and subtract, then

$$\frac{a_1b_2 - a_2b_1}{x} = c_1b_2 - c_2b_1 \text{ or } x = \frac{a_1b_2 - a_2b_1}{c_1b_2 - c_2b_1}.$$

18. $x(a - b) + ay = ab - b^2$, $ax + y(a + b) = ab + b^2$. Multiply (1) by $a + b$ and (2) by a and subtract, then $-b^2x = -b^3 - ab^2$ or $x = b + a$.

19. Multiply (2) by 3 and subtract, then $\frac{4.25\,b}{y} = 21.25$ or $y = \frac{1}{5}b$.

20. Multiply (2) by c and subtract and the result follows.

21. Multiply (1) by q and (2) by b and add.

22. Eliminate x from (1) and (2) and $y(a^2 - b^2) + acz = a^2c$. Now eliminate z from this equation and (3) and $y = 0$.

23. Solving (1) and (2) we find $x = a$, $y = b$. Now substitute in (3).

Exercise 103 — Page 214

1. 1. **2.** 4. **3.** 1, 1. **4.** 7. **5.** 9. **6.** $2\frac{1}{4}$. **7.** 30. **8.** $22\frac{1}{3}$. **9.** 17. **10.** $-\frac{8}{9}$. **11.** $a + b$. **12.** 1. **13.** 4. **14.** 17. **15.** $9\,a$, $-8\,b$. **16.** Cross multiply and $2\,bcx - 2\,adx = 0$ or $x = 0$.

17. $\frac{25}{x} + \frac{6}{y} = 105$, $\frac{70}{x} + \frac{6}{y} = 180$. Subtract and $\frac{45}{x} = 75$ or $x = \frac{3}{5}$.

18. $\frac{x}{a-2\,b} - \frac{2\,x}{2\,a-b} = \frac{1}{2}$ or $\frac{3\,bx}{(a-2\,b)(2\,a-b)} = \frac{1}{2}$ or $x = \frac{(a-2\,b)(2\,a-b)}{6\,b}$.

19. Multiply (1) by b and (2) by a and add, then $y = b$, $x = a$.

20. $\frac{x}{x+b-a} = \frac{x-c}{x+b-c}$. Cross multiply and $x = \frac{ac-bc}{a}$.

21. $\frac{-1}{(x+3)(x+4)} = \frac{-1}{(x+6)(x+7)}$ or $x^2+7\,x+12 = x^2+13\,x+42$; $x = -5$.

22. $\frac{5\,x-10}{x^2-100} = \frac{5}{x-2}$ or $5\,x^2 - 500 = 5\,x^2 - 20\,x + 20$; $x = 26$.

23. $\frac{x-1}{x-2} - \frac{x-3}{x-4} = \frac{x-5}{x-6} - \frac{x-7}{x-8}$. Proceed as in Ex. 21 and $x = 5$.

24. $\frac{x+1}{3\,x-4} - \frac{8\,x-3}{5(3\,x-4)} = \frac{1}{5}$ or $\frac{8-3\,x}{15\,x-20} = \frac{1}{5}$ or $x = 2$.

29. $\frac{4}{x} + \frac{3}{y} = -7$, $\frac{3}{y} - \frac{2}{x} = -10$. Subtract and $x = 2$, $y = -\frac{1}{3}$.

32. If they are x and $1000 - x$, then $\frac{11}{200}x - \frac{13}{200}(1000 - x) = \frac{16}{100}$.

33. If it is $\frac{x}{y}$, then $\frac{x+4}{y} = \frac{7}{8}$, $\frac{x}{y+4} = \frac{3}{4}$, then $x = 45$, $y = 56$.

34. Then $x(a - c) = d - b$. To find x we divide each side by $a - c$ but we can do this only if we assume that $a - c$ is not equal to zero.

35. If a is the greater and $1 - a$ the less, then $a^2 - (1 - a)^2 + 1 = 2\,a$.

37. Let $a = 7\,x$, $b = 4\,x$, $c = \frac{14\,x}{5}$ then $7\,x + 4\,x + \frac{14\,x}{5} = 3036$.

39. Solving (1) and (2) $x = a + b$, $y = a - b$. Now substitute in (3).

40. If it is x miles, then $\frac{x}{2\frac{1}{2}} + \frac{56-x}{3\frac{1}{2}} = 20$, then $x = 35$.

41. If the house cost \$$4x$, the farm cost \$$15x$. Selling price of the house $= \frac{9}{10} \times 4x$ and of farm is $\frac{15}{14} \times 15x$ or $\frac{18}{5}x + \frac{225}{14}x = 2754$.

42. Let the original pop. be $41x$, then the foreign was x and the native $40x$. Then $x - 1160 + \frac{28}{25} \times 40x = \frac{111}{100} \times 41x$ or $x = 4000$.

Exercise 104 — Page 216

1. $a^2b^2c^2$. **2.** $x^2 + 2x + 1$. **3.** x^2+2x+1. **4.** $4a^2-12ab+9b^2$. **5.** $a^4 + 2a^2 + 1$. **6.** $x^4 - 2x^3 + x^2$. **7.** $a^2+b^2+c^2+2ab+2bc+2ca$. **8.** $a^2 + b^2 + 1 + 2ab - 2a - 2b$. **9.** $4a^2 + b^2 + c^2 + 4ab - 4ac - 2bc$. **10.** $4xy$. **11.** $\frac{1}{2}ab^2$. **12.** $x + a$. **13.** $a - 1$. **14.** $2a - 3b$. **15.** $3x-5y$. **16.** $x+\frac{1}{2}$. **17.** a^2+a. **18.** $4x^2-6$. **19.** $x+y+z$. **20.** $a - b - c$. **21.** $2a + 3b - 1$.

Exercise 105 — Page 218

1.
$$\begin{array}{r|l} & x^2 + 12x + 36 \;\big|\; \underline{x + 6} \\ & x^2 \\ 2x + 6 & 12x + 36 \\ & 12x + 36 \end{array}$$

2.
$$\begin{array}{r|l} & 9a^2 - 6a + 1 \;\big|\; \underline{3a - 1} \\ & 9a^2 \\ 6a - 1 & -6a + 1 \\ & -6a + 1 \end{array}$$

3.
$$\begin{array}{r|l} & 9x^2 + 24xy + 16y^2 \;\big|\; \underline{3x + 4y} \\ & 9x^2 \\ 6x + 4y & 24xy + 16y^2 \\ & 24xy + 16y^2 \end{array}$$

4.
$$\begin{array}{r|l} & 25x^2 - 10xy + y^2 \;\big|\; \underline{5x - y} \\ & 25x^2 \\ 10x - y & -10xy + y^2 \\ & -10xy + y^2 \end{array}$$

5.
$$\begin{array}{r|l} & 1 - 18ab + 81a^2b^2 \;\big|\; \underline{1 - 9ab} \\ & 1 \\ 2 - 9ab & -18ab + 81a^2b^2 \\ & -18ab + 81a^2b^2 \end{array}$$

6.
$$\begin{array}{r|l} & 49a^4 - 28a^2b^2 + 4b^4 \;\big|\; \underline{7a^2 - 2b^2} \\ & 49a^4 \\ 14a^2 - 2b^2 & -28a^2b^2 + 4b^4 \\ & -28a^2b^2 + 4b^4 \end{array}$$

7.
$$\begin{array}{r|l} & a^4 + 2a^3 - 3a^2 - 4a + 4 \;\big|\; \underline{a^2 + a - 2} \\ & a^4 \\ 2a^2 + a & +2a^3 - 3a^2 - 4a + 4 \\ & +2a^3 + a^2 \\ 2a^2 + 2a - 2 & -4a^2 - 4a + 4 \\ & -4a^2 - 4a + 4 \end{array}$$

8.
$$\begin{array}{r|l} & 4x^4 + 4x^3 + 5x^2 + 2x + 1 \;\big|\; \underline{2x^2 + x + 1} \\ & 4x^4 \\ 4x^2 + x & 4x^3 + 5x^2 + 2x + 1 \\ & 4x^3 + x^2 \\ 4x^2 + 2x + 1 & 4x^2 + 2x + 1 \\ & 4x^2 + 2x + 1 \end{array}$$

9.

$$\begin{array}{rl}
 & x^4 - 6x^3 + 17x^2 - 24x + 16 \;|\; \underline{x^2 - 3x + 4} \\
 & \underline{x^4} \\
\underline{2x^2 - 3x} \;| & -6x^3 + 17x^2 - 24x + 16 \\
 & \underline{-6x^3 + 9x^2} \\
\underline{2x^2 - 6x + 4} \;| & 8x^2 - 24x + 16 \\
 & \underline{8x^2 - 24x + 16}
\end{array}$$

10.

$$\begin{array}{rl}
 & 9a^4 - 12a^3 + 34a^2 - 20a + 25 \;|\; \underline{3a^2 - 2a + 5} \\
 & \underline{9a^4} \\
\underline{6a^2 - 2a} \;| & -12a^3 + 34a^2 - 20a + 25 \\
 & \underline{-12a^3 + 4a^2} \\
\underline{6a^2 - 4a + 5} \;| & 30a^2 - 20a + 25 \\
 & \underline{30a^2 - 20a + 25}
\end{array}$$

11.

$$\begin{array}{rl}
 & a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \;|\; \underline{a^2 - 2ab + b^2} \\
 & \underline{a^4} \\
\underline{2a^2 - 2ab} \;| & -4a^3b + 6a^2b^2 - 4ab^3 + b^4 \\
 & \underline{-4a^3b + 4a^2b^2} \\
\underline{2a^2 - 4ab + b^2} \;| & 2a^2b^2 - 4ab^3 + b^4 \\
 & \underline{2a^2b^2 - 4ab^3 + b^4}
\end{array}$$

12.

$$\begin{array}{rl}
 & a^4 - 4a^3 \qquad + 8a + 4 \;|\; \underline{a^2 - 2a - 2} \\
 & \underline{a^4} \\
\underline{2a^2 - 2a} \;| & -4a^3 \qquad + 8a + 4 \\
 & \underline{-4a^3 + 4a^2} \\
\underline{2a^2 - 4a - 2} \;| & -4a^2 + 8a + 4 \\
 & \underline{-4a^2 + 8a + 4}
\end{array}$$

13.

$$\begin{array}{rl}
 & 9a^4 + 12a^3b + 34a^2b^2 + 20ab^3 + 25b^4 \;|\; \underline{3a^2 + 2ab + 5b^2} \\
 & \underline{9a^4} \\
\underline{6a^2 + 2ab} \;| & 12a^3b + 34a^2b^2 + 20ab^3 + 25b^4 \\
 & \underline{12a^3b + 4a^2b^2} \\
\underline{6a^2 + 4ab + 5b^2} \;| & 30a^2b^2 + 20ab^3 + 25b^4 \\
 & \underline{30a^2b^2 + 20ab^3 + 25b^4}
\end{array}$$

14.

$$\begin{array}{rl}
 & x^6 - 4x^5 \qquad + 6x^3 + 8x^2 + 4x + 1 \;|\; \underline{x^3 - 2x^2 - 2x - 1} \\
 & \underline{x^6} \\
\underline{2x^3 - 2x^2} \;| & -4x^5 \qquad + 6x^3 + 8x^2 + 4x + 1 \\
 & \underline{-4x^5 + 4x^4} \\
\underline{2x^3 - 4x^2 - 2x} \;| & -4x^4 + 6x^3 + 8x^2 + 4x + 1 \\
 & \underline{-4x^4 + 8x^3 + 4x^2} \\
\underline{2x^3 - 4x^2 - 4x - 1} \;| & -2x^3 + 4x^2 + 4x + 1 \\
 & \underline{-2x^3 + 4x^2 + 4x + 1}
\end{array}$$

15.

$$\begin{array}{r|l}
 & x^4 - 2x^3 + 2x^2 - x + \tfrac{1}{4} \;\big|\; \underline{x^2 - x + \tfrac{1}{2}} \\
 & \underline{x^4} \\
2x^2 - x & -2x^3 + 2x^2 - x + \tfrac{1}{4} \\
 & \underline{-2x^3 + x^2} \\
2x^2 - 2x + \tfrac{1}{2} & x^2 - x + \tfrac{1}{4} \\
 & \underline{x^2 - x + \tfrac{1}{4}}
\end{array}$$

16.

$$\begin{array}{r|l}
 & \frac{a^4}{b^4} + \frac{4a^3}{b^3} + \frac{2a^2}{b^2} - \frac{4a}{b} + 1 \;\Big|\; \underline{\frac{a^2}{b^2} + \frac{2a}{b} - 1} \\
 & \underline{\frac{a^4}{b^4}} \\
\frac{2a^2}{b^2} + \frac{2a}{b} & \frac{4a^3}{b^3} + \frac{2a^2}{b^2} - \frac{4a}{b} + 1 \\
 & \underline{\frac{4a^3}{b^3} + \frac{4a^2}{b^2}} \\
\frac{2a^2}{b^2} + \frac{4a}{b} - 1 & -\frac{2a^2}{b^2} - \frac{4a}{b} + 1 \\
 & \underline{-\frac{2a^2}{b^2} - \frac{4a}{b} + 1}
\end{array}$$

17.

$$\begin{array}{r|l}
 & a^2 - 4ab + 6ac + 4b^2 - 12bc + 9c^2 \;\big|\; \underline{a - 2b + 3c} \\
 & \underline{a^2} \\
2a - 2b & -4ab + 6ac + 4b^2 - 12bc + 9c^2 \\
 & \underline{-4ab + 4b^2} \\
2a - 4b + 3c & +6ac - 12bc + 9c^2 \\
 & \underline{+6ac - 12bc + 9c^2}
\end{array}$$

18. The expression $= a^4 + 6a^3 + 11a^2 + 6a + 1 = (a^2 + 3a + 1)^2$.

19. Finding the square root in the usual way we get $x^2 + 2x + 1$ with a remainder $6 - x$. If it is a perfect square the remainder is 0 and then $x=6$. When 6 is substituted for x we get 2401, which is the square of 49.

20. In finding the root the second term is $-4x$, so $m = -4$.

21. $(x+1)(x+2)(x+2)(x+3)(x+3)(x+1) = \{(x+1)(x+2)(x+3)\}^2$.

22.

$$\begin{array}{r|l}
 & 1 - 2x - 3x^2 \;\big|\; \underline{1 - x - 2x^2} \\
 & \underline{1} \\
2 - x & -2x - 3x^2 \\
 & \underline{-2x + x^2} \\
2 - 2x - 2x^2 & -4x^2 \\
 & \underline{-4x^2 + 4x^3 + 4x^4}
\end{array}
\qquad
\begin{array}{r|l}
 & 4 - 12x \;\big|\; \underline{2 - 3x - 4\tfrac{1}{2}x^2} \\
 & \underline{4} \\
4 - 3x & -12x \\
 & \underline{-12x + 9x^2} \\
2 - 6x - 4\tfrac{1}{2}x^2 & -9x^2 \\
 & \underline{-9x^2 + 27x^3 + 20\tfrac{1}{4}x^4}
\end{array}$$

23. The square root of the expression is $2x^2 + x + 2$ and therefore the square root of 44944 is 212.

Exercise 106 — Page 221

1. The first term in the square root is x^2, the trial divisor is $2x^2$, therefore the second term is $+x$. The square root then is x^2+x+1 or x^2+x-1, and the term $-2x$ shows that it must be x^2+x-1.

2. x^2-2x+1. **3.** a^2-3a-2. **4.** x^2+4x-2. **5.** $3a^2-a+2$.

6. $x^2+3xy-y^2$. **7.** $2x^2+5x-7$. **8.** $1-5x+x^2$.

9. $3x^2-5x+7$. **10.** a^6-4a^3+1. **11.** $x^2+x-\frac{1}{2}$.

12. $x^2-x+\frac{1}{4}$. **13.** $a^2-3+\dfrac{1}{a^2}$. **14.** $\dfrac{2x^2}{y^2}-\dfrac{x}{y}-1$.

15. $\frac{1}{2}x^2-3x+\frac{1}{3}$. **16.** $\dfrac{3a^2}{5}+\dfrac{2a}{3}+1$. **17.** $x-1+\dfrac{2}{x}$.

18. $\dfrac{x}{y}-1+\dfrac{y}{x}$. **20.** $x^4-6ax^3+11a^2x^2-6a^3x+a^4=(x^2-3ax+a^2)^2$.

21. The expression $=a^4+2a^2b^2+b^4=(a^2+b^2)^2$.

22. The expression $=a^4+2a^2b^2+b^4=(a^2+b^2)^2$.

23. $x^4+4x^2+6+\dfrac{4}{x^2}+\dfrac{1}{x^4}=\left(x^2+2+\dfrac{1}{x^2}\right)^2$.

24. Take the square root in the usual way and it will be seen that $a=-6$ to be a perfect square.

25. If they are a, $a+1$, then $a^2+(a+1)^2+a^2(a+1)^2$
$$=a^4+2a^3+3a^2+2a+1=(a^2+a+1)^2.$$

26. It must be the square of $2x^2+3xy+y^2$.

27. $m^2+4=\left(x-\dfrac{1}{x}\right)^2+4=\left(x+\dfrac{1}{x}\right)^2$ and $n^2+4=\left(y+\dfrac{1}{y}\right)^2$.

The first side $=\left(x-\dfrac{1}{x}\right)\left(y-\dfrac{1}{y}\right)+\left(x+\dfrac{1}{x}\right)\left(y+\dfrac{1}{y}\right)=2xy+\dfrac{2}{xy}$.

28. The square root $=2x^2+2x+1=221$ when $x=10$. The given expression $=48{,}841$ when $x=10$, but $221^2=48{,}841$.

Exercise 107 — Page 223

1. $-\frac{1}{27}$. **2.** $-8a^3$. **3.** $-27a^3b^6$. **4.** $-x^6y^3z^9$. **5.** $x^3+3x^2y+3xy^2+y^3$. **6.** $x^3-3x^2y+3xy^2-y^3$. **9.** x^3+3x^2+3x+1. **10.** x^3-3x^2+3x-1. **11.** $a^6+3a^4b+3a^2b^2+b^3$. **13.** $x^3+9x^2+27x+27$. **14.** $8x^3-12x^2y+6xy^2-y^3$. **15.** $8a^3+36a^2b+54ab^2+27b^3$. **16.** $1-6a+12a^2-8a^3$. **17.** $a^3-12a^2b+48ab^2-64b^3$. **18.** $1-3a^2+3a^4-a^6$. **19.** $a^3+b^3-c^3+3a^2b+3ab^2-3a^2c+3ac^2-3b^2c+3bc^2-6abc$. **20.** $a^3-b^3-c^3-3a^2b+3ab^2-3a^2c+3ac^2-3b^2c-3bc^2+6abc$. **22.** $(x-y)^3=x^3-y^3-3xy(x-y)$.

24. Each side $=3(ab^2-a^2b+bc^2-b^2c+ca^2-c^2a)$.

25. If they are x and $x+1$, then $(x+1)^3-x^3=3x^2+3x+1$.

26. $(y+z)^3-y^3-z^3=3\,y^2z+3\,yz^2=3\,yz(y+z)\;=3\,xyz.$

27. If they are x and $x+3$, then $(x+3)^3-x^3-9\,x(x+3)=27.$

28. Let the integers be $x-1$, x, and $x+1$. Then $(x-1)x\,(x+1)+x=x^3-x+x=x^3.$

Exercise 108 — Page 225

1. $-4.$ **2.** $3\,a.$ **3.** $-5\,ab.$ **4.** $-2(a-b).$ **5.** $x+1.$ **6.** $x-y.$ **7.** $a+2.$ **8.** $2\,x-1.$ **9.** $xy+1.$ **10.** $4\,a-3.$ **11.** $5\,x-1.$ **12.** $3\,x-y.$ **13.** $\frac{1}{2}x-1.$ **14.** $\frac{1}{3}x+2.$ **15.** $m-\frac{3}{m}.$

16. $\frac{x^2}{y}-2\,y^2.$

17. The first term is x^2 and the last is $+1$. The trial divisor is $3\,x^4$, therefore the second term is $3\,x^5\div 3\,x^4=x$. Then the cube root is x^2+x+1. When $x=1$ the expression $=27$ and its cube root is 3, which is the value of x^2+x+1 when $x=1$.

18. The first term is 1 and the last is $+3\,x^2$. The trial divisor is 3 so the second term is $-6\,x\div 3=-2\,x$.

19. The first term is $\frac{x}{3}$ and the last is $\frac{3}{x}$. The trial divisor is $3\times\left(\frac{x}{3}\right)^2$ or $\frac{x^2}{3}$, therefore the second term is $-\frac{x^2}{3}\div\frac{x^2}{3}=-1$.

20. First term $=3\,a^2$, second $=-108\,a^5\div 27\,a^4$, last $=+1$.

21. $1+6\,x^2+9\,x^4-9\,x^2-6\,x^4-x^6=1-3\,x^2+3\,x^4-x^6=(1-x^2)^3.$

22. From the first two terms we see that it must be the cube of $x+c$. Then $x^3+3\,cx^2+3\,c^2x+c^3=x^3+3\,cx^2+2\,c^2x+5\,c^3$.

23. The square root is $x^2-2\,x+1$ and the fourth root is $x-1$.

24. The square root is $a^2-6\,a+9$, the fourth root is $a-3$.

25. The cube root is $x^2-4\,x+4$, the sixth root is $x-2$.

Exercise 109 — Page 226

1. First term $=3\,x^2$, second $=-24\,x^3y\div 6\,x^2=-4\,xy$, last $=+2\,y^2$.

2. First term $=x^3$, second $=+2\,x^2$. Reading from the end the last term is 1 and the next $-3\,x$, so that the square root is either $x^3+2\,x^2-3\,x+1$ or $x^3+2\,x^2+3\,x-1$. By trial it is easily seen to be the former.

3. First two terms are $x^6+3\,x^4$, the last two are $2-2\,x^2$ or $-2+2\,x^2$.

4. First term $=\frac{1}{2}\,x^2$, second $=-\frac{1}{3}\,x^3\div x^2=-\frac{1}{3}\,x$, last $=+1$.

5. First term $=5\,x^2$, second $=-20\,ax^3\div 10\,x^2=-2\,ax$, last $=-3\,a^2$.

6. $4\,x^4+4\,x^2(3\,a+7)+(3\,a+7)^2=(2\,x^2+3\,a+7)^2.$

7. $(x+3)(x+2)(x+3)(x+4)(x+4)(x+2)=(x+2)^2(x+3)^2(x+4)^2.$

8. $(x+1)(2x-3)(x+1)(x-5)(x-5)(2x-3)$
$$=(x+1)^2(x-5)^2(2x-3)^2.$$

9. First two terms are $2x^2-5x$, last two are $\frac{3}{x}-2$ or $-\frac{3}{x}+2$.

10. First term $=3$, last $=-5x$ and the cube root $=3-5x$.

11. First term $=2x^2$, second $=-12x^5 \div 12x^4=-x$, last $=+1$.

12. It is evidently the cube of $(a-b)+b=a$.

13.
$$\begin{array}{l|l} 1-2x & \underline{1-x-\frac{1}{2}x^2} \\ 1 & \\ \hline \underline{2-x} & -2x \\ & \underline{-2x+x^2} \\ \underline{2-2x-\frac{1}{2}x^2} & -x^2 \end{array} \qquad \begin{array}{l|l} 4+x & \underline{2+\frac{1}{4}x-\frac{1}{64}x^2} \\ 4 & \\ \hline \underline{4+\frac{1}{4}x} & +x \\ & \underline{+x+\frac{1}{16}x^2} \\ \underline{4+\frac{1}{2}x-\frac{1}{64}x^2} & -\frac{1}{16}x^2 \end{array}$$

14. If it is the square of $x-y$, then $x^2-2xy+y^2=x^2-2(a-y)x+y^2$ or $y=a-y$ or $y=\frac{1}{2}a$. When $y=\frac{1}{2}a$ the given expression $=x^2-ax+\frac{1}{4}a^2=(x-\frac{1}{2}a)^2$.

15. First two terms in square root $=7x^2-2x$ and the last $=-\frac{3}{2}$.

16. $a(a+1)(a+2)(a+3)+1=a^4+6a^3+11a^2+6a+1$
$$=(a^2+3a+1)^2.$$

17. Root $=a^2+2a+1$. Put $a=10$ then $\sqrt{14641}=10^2+2\cdot 10+1=121$.

18. It is evidently $\{(a+b)+(a-b)\}^3=(2a)^3=8a^3$.

19. $(b+1)^3-b^3-1=3b^2+3b=3b(b+1)=3ab$.

20. $a(a+2)(a+4)(a+6)+16=a^4+12a^3+44a^2+48a+16=(a^2+6a+4)^2$. Since a is an even integer then $\frac{1}{2}a$ is any integer. Now substitute $\frac{1}{2}a$ for a in Ex. 16 and simplify the factors.

21. The first $=(a-b+b-c+c-a)^2=0$, second $=(2x-y-2x-y)^3=-8y^3$.

22. $\{2a(a+1)\}^2+(a+a+1)^2=(2a^2+2a)^2+(2a+1)^2=4a^4+8a^3+8a^2+4a+1=(2a^2+2a+1)^2$.

23. $x^3-y^3=(x-y)^3+3xy(x-y)$.

24. The first two terms $=2x^3-3x^2$, the last two $=-2+x$.

25. This is the same as Ex. 20, as the solution will hold whether a is odd or even.

26. Let them be $a-1$, a, $a+1$, then $(a-1)^3+a^3+(a+1)^3=3a^3+6a=3a(a-1)(a+1)+9a$.

27. The expression $=(4x-1+2x-3)^3=(6x-4)^3$.

28. Using the formal method we find the square root to be $2x^3+3x^2-x+1$. Squaring this the remaining terms are found.

Exercise 110 — Page 230

1. $\sqrt{6}$. **2.** 5. **3.** 4. **4.** 42. **5.** $\sqrt{30}$. **6.** 2. **7.** $\sqrt{3}$. **8.** abc. **9.** $\sqrt{12}$. **10.** $\sqrt{18}$. **11.** $\sqrt{125}$. **12.** $\sqrt{a^2b}$. **13.** $\sqrt{3\,a^2}$. **14.** $\sqrt{ab}$. **15.** $\sqrt{9\,a-9\,b}$. **16.** $\sqrt{a^2-b^2}$. **17.** $2\sqrt{2}$. **18.** $2\sqrt{3}$. **19.** $2\sqrt{5}$. **20.** $5\sqrt{3}$. **21.** $3\sqrt{3}$. **22.** $2\sqrt{14}$. **23.** $9\sqrt{2}$, **24.** $a\sqrt{2}$. **25.** $10\,x\sqrt{10\,x}$. **26.** $2\sqrt{2}$. **27.** $(a-b)\sqrt{a-b}$. **28.** $\sqrt{b}$. **29.** $\pm\sqrt{2}$, ± 3, $\pm 3\sqrt{2}$. **30.** See Art. 159.

31. $\sqrt{8}=2.828$, $\sqrt{2}=1.414$. **32.** If the hyp. $=x$, then $x^2=2^2+3^2$.

33. Make the sides 1 in. and 3 in., then the hyp. $=\sqrt{10}$ in.

34. $3\frac{1}{7}\,r^2=66$ or $r^2=21$ or $r=\sqrt{21}$.

35. If they are x and $2\,x$, then $x^2+4\,x^2=40$ or $x^2=8$, $x=2\sqrt{2}$.

36. If the side is x, then $x^2+x^2=100$ or $x^2=50$, $x=5\sqrt{2}$.

37. If the sides are x and $3\,x$, then $3\,x^2=96$, $x=\sqrt{32}=4\sqrt{2}$.

Exercise 111 — Page 231

1. $8\sqrt{2}$. **2.** $2\sqrt{7}$. **3.** $5\sqrt{a}$. **4.** $6\sqrt{x}$. **5.** $3\sqrt{2}$. **6.** $3\sqrt{3}$. **7.** $3\sqrt{2}-2\sqrt{2}=\sqrt{2}$. **8.** $2\sqrt{a}+3\sqrt{a}=5\sqrt{a}$. **9.** $5\sqrt{3}+2\sqrt{3}+3\sqrt{3}$. **10.** $6\sqrt{2}+6\sqrt{2}-5\sqrt{2}$. **11.** $3\sqrt{5}-2\sqrt{5}+4\sqrt{5}$. **12.** $6\sqrt{7}-10\sqrt{7}+\sqrt{7}$. **13.** $32\sqrt{2}+20\sqrt{2}-45\sqrt{2}$. **14.** $20\sqrt{11}-12\sqrt{11}$. **15.** $3\sqrt{5}+2\sqrt{5}-4\sqrt{5}+6\sqrt{5}$. **16.** $6\sqrt{2}+7\sqrt{2}-8\sqrt{2}-4\sqrt{2}-5\sqrt{2}$. **17.** $5\sqrt{3}$. **18.** $3\sqrt{7}$. **19.** $3\sqrt{15}$. **20.** $7\sqrt{3}-4\sqrt{3}$. **21.** $8\sqrt{2}-9\sqrt{2}$. **22.** $2\sqrt{14}+6\sqrt{2}+3\sqrt{10}$. **23.** $\pm\sqrt{37}$. **24.** $\pm\sqrt{15}$. **25.** $\pm\sqrt{46}$. **26.** $\pm\sqrt{42}$. **27.** $\pm 2\sqrt{23}$. **28.** $\pm 2\sqrt{47}$. **29.** $3\frac{1}{7}\,r^2=176$ or $r^2=56$, $r=2\sqrt{14}$.

Exercise 112 — Page 234

1. $6\sqrt{15}$. **2.** $30\sqrt{6}$. **3.** $ab\sqrt{ab}$. **4.** $3\sqrt{30}$. **5.** $12\times 2=24$. **6.** $2+\sqrt{2}$. **7.** $\sqrt{6}+\sqrt{10}$. **8.** $\sqrt{ac}+\sqrt{bc}-\sqrt{c}$. **9.** $3-2=1$. **10.** $10-9=1$. **11.** $x-y$. **12.** $8-3=5$. **13.** $12\sqrt{12}=24\sqrt{3}$. **14.** $6\sqrt{28}=12\sqrt{7}$. **15.** $3+2+2\sqrt{6}$. **16.** $20+7-4\sqrt{35}$. **17.** $18+12+12\sqrt{6}$.

19.
$$\begin{array}{l}
4+3\sqrt{2}\\
\underline{5-3\sqrt{2}}\\
20+15\sqrt{2}\\
\underline{\quad -12\sqrt{2}-18}\\
20+3\sqrt{2}-18
\end{array}$$

20.
$$\begin{array}{l}
3\sqrt{2}+2\sqrt{3}\\
\underline{5\sqrt{2}-3\sqrt{3}}\\
30+10\sqrt{6}\\
\underline{\quad -9\sqrt{6}-18}\\
30+\sqrt{6}-18
\end{array}$$

21.
$$\begin{array}{l}
3\sqrt{5}-4\sqrt{2}\\
\underline{2\sqrt{5}+3\sqrt{2}}\\
30-8\sqrt{10}\\
\underline{\quad +9\sqrt{10}-24}\\
30+\sqrt{10}-24
\end{array}$$

23. $\{(\sqrt{5}+\sqrt{3})+\sqrt{2}\}\{(\sqrt{5}+\sqrt{3})-\sqrt{2}\}=(\sqrt{5}+\sqrt{3})^2-2=5+3+2\sqrt{15}-2$. **24.** $\{\sqrt{7}+(2\sqrt{2}-\sqrt{3})\}\{\sqrt{7}-(2\sqrt{2}-\sqrt{3})\}=7-(2\sqrt{2}-\sqrt{3})^2=7-8-3+4\sqrt{6}$. **27.** $3+2+1+2\sqrt{6}+2\sqrt{3}+2\sqrt{2}$.
28. $5+8+3+4\sqrt{10}-2\sqrt{15}-4\sqrt{6}$. **29.** $a+b+a-b+2\sqrt{a^2-b^2}$.
30. $9(x-y)+4(x+y)-12\sqrt{x^2-y^2}$. **31.** $(36-12)-(25-2)$.

32. $(6-2\sqrt{6}+2\sqrt{3}-2\sqrt{2})+(6+2\sqrt{6}-2\sqrt{3}-2\sqrt{2})$.

33. $(5\sqrt{2}-3\sqrt{2}+6\sqrt{2}+4\sqrt{2})\times\frac{1}{2}\sqrt{3}=12\sqrt{2}\times\frac{1}{2}\sqrt{3}=6\sqrt{6}$.

34. $2(36+\sqrt{6}-12)+(40-2\sqrt{6}-18)=48+2\sqrt{6}+22-2\sqrt{6}$.

35. Product of first two $=4+\sqrt{6}$, of the last two $=15-5\sqrt{6}$.
Product of $4+\sqrt{6}$ and $15-5\sqrt{6}=60-5\sqrt{6}-30=30-5\sqrt{6}$.

36. $(\sqrt{10}+\sqrt{5})^2=10+5+2\sqrt{50}=15+2\sqrt{50}$, $(\sqrt{8}+\sqrt{7})^2=15+2\sqrt{56}$.

37. The product $=45+4\sqrt{21}-21=24+4\sqrt{21}=24+4\times 4.5826=42.3304$, which lies between 42 and 43.

38. The area $=(5+\sqrt{2})(10-2\sqrt{2})=50-4=46$.

39. If x is the hyp., then $x^2=(7+4\sqrt{2})^2+(7-4\sqrt{2})^2=49+56\sqrt{2}+32+49-56\sqrt{2}+32=162$. Then $x=\sqrt{162}=9\sqrt{2}$.

40. The area $=\frac{1}{2}(2\sqrt{3}+3\sqrt{2})(3\sqrt{3}+2\sqrt{2})=\frac{1}{2}(30+13\sqrt{6})=15+\frac{13}{2}\sqrt{6}=15+15.92=30.92$.

Exercise 113 — Page 236

1. $3\sqrt{9}=9$. **2.** $\sqrt{4}=2$. **3.** $6\sqrt{2}\div 6\sqrt{2}=1$. **4.** $\sqrt{bc}$.
5. $\sqrt{6}+2$. **6.** $\sqrt{b}+\sqrt{c}$. **7.** $\frac{2\sqrt{3}}{3}$. **8.** $2\sqrt{5}$. **9.** $\frac{\sqrt{15}}{3}$.
10. $\frac{a\sqrt{b}}{b}$. **11.** $\frac{\sqrt{30}}{2}$. **12.** $\sqrt{2}+1$. **13.** $\frac{2(7+4\sqrt{3})}{49-48}$.
14. $\frac{12(3\sqrt{2}+2\sqrt{3})}{18-12}$. **15.** $\frac{(\sqrt{3}+\sqrt{2})^2}{3-2}$. **16.** $\frac{\sqrt{a}(\sqrt{a}-\sqrt{b})}{a-b}$.
17. $\frac{(5\sqrt{3}-3\sqrt{5})(\sqrt{5}+\sqrt{3})}{5-3}=\sqrt{15}$. **18.** $\frac{(\sqrt{7}+\sqrt{2})(9-2\sqrt{14})}{81-56}=\frac{5\sqrt{7}-5\sqrt{2}}{25}$. **19.** $\frac{\sqrt{3}}{3}=\frac{1.732}{3}=.577$. **20.** $\frac{5}{6}\sqrt{18}$. **21.** $\frac{\sqrt{6}}{3}$.
22. $\sqrt{3}-\sqrt{2}$. **23.** $\frac{17(3\sqrt{7}-2\sqrt{3})}{63-12}=\frac{3\sqrt{7}-2\sqrt{3}}{3}$. **24.** $\frac{(\sqrt{7}-\sqrt{5})^2}{7-5}=6-\sqrt{35}$. **25.** $\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$. **26.** $\frac{6\sqrt{7}}{3\sqrt{35}}=\frac{6}{3\sqrt{5}}=\frac{2\sqrt{5}}{5}$. **27.** $7-4\sqrt{3}$.
28. $x=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$. **29.** $x=\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$. **30.** $x=\frac{\sqrt{2}+1}{\sqrt{3}}=\frac{\sqrt{6}+\sqrt{3}}{3}$.
31. $x=\frac{1}{\sqrt{3}-\sqrt{2}}=\sqrt{3}+\sqrt{2}$. **32.** $x=\frac{5-\sqrt{5}}{\sqrt{5}-2}=5+3\sqrt{5}$.

33. $x^2 = \dfrac{2(\sqrt{3}+1)}{\sqrt{3}-1} = (\sqrt{3}+1)^2$. Then $x = \pm(\sqrt{3}+1)$.

34. The base $= \dfrac{4}{\sqrt{5}+\sqrt{3}} = 2(\sqrt{5}-\sqrt{3})$.

35. The denominator $= 4\sqrt{2} + 2\sqrt{5} - 3\sqrt{2} - \sqrt{5} = \sqrt{2} + \sqrt{5}$.

The fraction $= \dfrac{2+\sqrt{10}}{\sqrt{5}+\sqrt{2}} = \dfrac{(2+\sqrt{10})(\sqrt{5}-\sqrt{2})}{3} = \dfrac{3\sqrt{2}}{3} = \sqrt{2}$.

Exercise 114 — Page 237

1. $\sqrt{x} = 3,\ x = 9$. **2.** $\sqrt{x} = 9,\ x = 81$. **3.** $\sqrt{x} = 5,\ x = 25$. **4.** $\sqrt{x} = 2, x = 4$. **5.** $\sqrt{x} = \frac{1}{4}\sqrt{20},\ x = 1\frac{1}{4}$. **6.** $\sqrt{x} = a + b$, $x = (a+b)^2$. **7.** $\sqrt{x} = n - m,\ x = (n-m)^2$. **8.** $\sqrt{x-4} = 4$, $x - 4 = 16,\ x = 20$. **9.** Squaring, $x^2 + 9 = 81 - 18x + x^2,\ 18x = 72$, $x = 4$. **10.** $x^2 + 11x + 3 = x^2 + 10x + 25,\ x = 22$.

11. $9x^2 - 11x - 5 = 9x^2 - 12x + 4,\ x = 9$.

12. $4x^2 - 10x + 4 = (2x-4)^2 = 4x^2 - 16x + 16,\ x = 2$.

13. $x + a^2 = (b-a)^2 = b^2 - 2ab + a^2,\ x = b^2 - 2ab$.

14. $x^2 - 2ax + a^2 + 2ab + b^2 = x^2 + a^2 + b^2 - 2ax + 2bx - 2ab$, $4ab = 2bx,\ x = 2a$.

Exercise 115 — Page 238

1. $2\sqrt{2} + 3\sqrt{2} + 7\sqrt{2}$. **2.** $10\sqrt{5} + 4\sqrt{5} - 2\sqrt{5}$.

3. $5\sqrt{3} + 9\sqrt{3} - 4\sqrt{3}$. **4.** $80 - 18 = 62$. **5.** $216 - 25 = 191$.

6. $6 - (\sqrt{2}+2)^2 = 6 - 2 - 4 - 4\sqrt{2} = -4\sqrt{2}$.

7. $(2\sqrt{2} + \sqrt{2} - 2)^2 = (3\sqrt{2} - 2)^2 = 18 + 4 - 12\sqrt{2}$.

8. $3 + 8 + 1 - 4\sqrt{6} - 2\sqrt{3} + 4\sqrt{2}$. **9.** $15\sqrt{3} \div 30\sqrt{3} = \frac{1}{2}$.

10. $\dfrac{\sqrt{5}-2}{\sqrt{5}+2} = \dfrac{(\sqrt{5}-2)^2}{1} = 9 - 4\sqrt{5}$. **11.** $(5\sqrt{5} + 3\sqrt{5}) \div 8\sqrt{5} = 1$.

12. $\dfrac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{13}-\sqrt{2}} = \dfrac{22}{\sqrt{13}-\sqrt{2}} = \dfrac{22(\sqrt{13}+\sqrt{2})}{13-2} = 2\sqrt{13} + 2\sqrt{2}$.

13. $(6\sqrt{2} + 2\sqrt{3} - \sqrt{2})(4\sqrt{2} - \sqrt{3} + 4\sqrt{2}) = (5\sqrt{2} + 2\sqrt{3})(8\sqrt{2} - \sqrt{3})$.

14. $\left(\sqrt{3} + \dfrac{1}{\sqrt{3}}\right)^2 = 3 + \frac{1}{3} + 2 = 5\frac{1}{3}$, $\left(\sqrt{2} + \dfrac{1}{\sqrt{2}}\right)^2 = 2 + \frac{1}{2} + 2 = 4\frac{1}{2}$.

16. See 36, page 234. **17.** The product $= 8\sqrt{6} - 18 = 1.596$.

18. $\frac{4\sqrt{2}}{2}$, $\frac{3\sqrt{6}}{6}$, $\frac{3\sqrt{5}}{2\sqrt{6}}=\frac{3\sqrt{30}}{12}$, $\frac{\sqrt{7}}{\sqrt{2}}=\frac{\sqrt{14}}{2}$, $\frac{(2\sqrt{6}-2)(3\sqrt{3}-\sqrt{2})}{27-2}$ $=\frac{20\sqrt{2}-10\sqrt{3}}{25}$. **19.** $x+3=16$, $x=13$. **20.** $3x-2=4(x-2)$, $x=6$. **21.** $x^2-5=(x-1)^2$, $x=3$. **22.** $2x+7=9x$, $x=1$.

23. $x^2-5x+11=(x+2)^2=x^2+4x+4$, $x=\frac{7}{9}$.

24. $x^2-2=1-2x+x^2$, $x=1\frac{1}{2}$. This value of x will not satisfy the equation. This is an impossible equation and has no root.

25. $x=\pm\sqrt{75}=\pm 5\sqrt{3}$, $x=\pm\sqrt{63}=\pm 3\sqrt{7}$, $x=\pm\sqrt{98}=\pm 7\sqrt{2}$,

$$x=\sqrt{5}\div\sqrt{3}=\tfrac{1}{3}\sqrt{15},\ x=\frac{\sqrt{3}}{\sqrt{2}+1}=\sqrt{3}(\sqrt{2}-1)=\sqrt{6}-\sqrt{3}.$$

26. $\frac{1}{3}\sqrt{6}$, $\frac{1}{5}\sqrt{5}$, $\sqrt{2}-1$, $5-3\sqrt{2}$, $\frac{1}{31}(26-11\sqrt{2})$.

27. $(2\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})(3\sqrt{3}-\sqrt{2})=(9+\sqrt{6})(3\sqrt{3}-\sqrt{2})$ $=25\sqrt{3}$.

28. If the hyp. $=x$, then $x^2=(\sqrt{3}+1)^2+(\sqrt{3}-1)^2=8$, $x=2\sqrt{2}$.

29. $\frac{\sqrt{5}-1}{\sqrt{5}-2}-\frac{\sqrt{5}-3}{\sqrt{5}+3}=\frac{3+\sqrt{5}}{1}-\frac{14-6\sqrt{5}}{-4}=3+\sqrt{5}+\frac{7-3\sqrt{5}}{2}$.

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}=(5+2\sqrt{6})-(5-2\sqrt{6})=4\sqrt{6}.$$

30. $\frac{4}{2\sqrt{3}}+\frac{2\sqrt{3}}{4}=\frac{2\sqrt{3}}{3}+\frac{\sqrt{3}}{2}=\frac{7\sqrt{3}}{6}=2.02$.

31. $6\sqrt{10}-3\sqrt{15}+15+8\sqrt{15}-6\sqrt{10}+10\sqrt{6}-10\sqrt{6}+15-5\sqrt{15}=30$.

32. Product $=\sqrt{(7\times 2\sqrt{6})(7-2\sqrt{6})}=\sqrt{25}=5$.

33. $\frac{16\sqrt{10}-25}{3\sqrt{5}-\sqrt{2}}=\frac{(16\sqrt{10}-25)(3\sqrt{5}+\sqrt{2})}{45-2}=\frac{215\sqrt{2}-43\sqrt{5}}{43}$

$=5\sqrt{2}-\sqrt{5}$.

Exercise 117 — Page 242

6. $2x^2+2=5x$, $2x^2-5x+2=0$, $a=2$, $b=-5$, $c=2$.

7. $32-4x=x^2$, $x^2+4x-32=0$, $a=1$, $b=4$, $c=-32$.

8. $6x^2-25x+25=x^2+2x-3$, $5x^2-27x+28=0$.

9. $3x^2+7x-40=5x^2-12x+4$, $2x^2-19x+44=0$.

10. $4x(x+2)-5x(x-1)=3(x-1)(x+2)$, $4x^2-10x-6=0$, $2x^2-5x-3=0$, $a=2$, $b=-5$, $c=-3$.

11. $(x+1)(x-1)+3x(x+2)=4(x+2)(x-1)$, $2x+7=0$. Here $a=0$, $b=2$, $c=7$ and the equation is not a quadratic.

12. $2x^2-(x-3)^2=x(x-3)$, $9x-9=0$, $x-1=0$.

Exercise 118 — Page 244

1. $2, -1\frac{5}{6}$. **2.** $3, 1\frac{1}{6}$. **3.** $\frac{5}{8}, -3$. **4.** $\frac{1}{4}, \frac{2}{3}$. **5.** $1, 9$. **6.** $2, \frac{1}{2}$. **7.** $4, -8$. **8.** $4, 1\frac{2}{5}$. **9.** $4, 5\frac{1}{2}$. **10.** $3, -\frac{1}{2}$. **11.** $-3\frac{1}{2}$. **12.** 1. **13.** 11. **14.** 13. **15.** 7. **16.** 8. **17.** 10. **18.** 94. **19.** 11. **20.** $(x-a)(x-2a)=0$, $x=a$ or $2a$. **21.** $(x-b)(x+b)=0$, $x=\pm b$. **22.** $(x-3m)(x+2m)=0$, $x=3m$ or $-2m$. **23.** $(x-a)(x-b)=0$, $x=a$ or b. **24.** $(x+2a)(x+2b)=0$, $x=-2a$ or $-2b$. **25.** $(ax-1)(2x+1)=0$, $x=\dfrac{1}{a}$ or $-\dfrac{1}{2}$. **26.** $x^2-ax-bx+ab=ab$ or $x(x-a-b)=0$, $x=0$ or $a+b$. **27.** $(x-a)(x+a)-(x-a)(b+c)=0$ or $(x-a)(x+a-b-c)=0$, $x=a$ or $b+c-a$.

Exercise 119 — Page 246

1. 1. **2.** 4. **3.** 25. **4.** 49. **5.** $2\frac{1}{4}$. **6.** $6\frac{1}{4}$. **7.** $4a^2$. **8.** $\frac{1}{16}$. **9.** $x^2+4x+4-81=(x+2)^2-9^2=(x+2+9)(x+2-9)=(x+11)(x-7)$. **10.** $x^2-54x+729-16=(x-27)^2-4^2=(x-23)(x-31)$.

11. $x^2-2x+1-900=(x-1)^2-30^2=(x+29)(x-31)$.

12. $x^2-x+\frac{1}{4}-1640\frac{1}{4}=(x-\frac{1}{2})^2-(\frac{81}{2})^2=(x+40)(x-41)$.

13. $x^2-\frac{11}{2}x+\frac{121}{16}-\frac{1}{16}=(x-\frac{11}{4})^2-(\frac{1}{4})^2=(x-\frac{5}{2})(x-3)$.

14. $3(x^2+\frac{16}{3}x-33)=3(x^2+\frac{16}{3}x+\frac{64}{9}-\frac{361}{9})=3\{(x+\frac{8}{3})^2-(\frac{19}{3})^2\}$ $=3(x+\frac{8}{3}+\frac{19}{3})(x+\frac{8}{3}-\frac{19}{3})=3(x+9)(x-\frac{11}{3})=(x+9)(3x-11)$.

15. $x^2+8x+16=25$ or $x+4=\pm 5$ or $x=1$ or -9.

16. $x^2-6x+9=16$ or $x-3=\pm 4$ or $x=-1$ or 7.

17. $x^2-10x+25=-9+25$ or $x^2-10x+25=16$ or $x-5=\pm 4$, $x=9$ or 1.

18. $x^2-9x=-18$ or $x^2-9x+\frac{81}{4}=\frac{9}{4}$ or $x-\frac{9}{2}=\pm\frac{3}{2}$ or $x=6$ or 3.

19. $x^2+7x=-10$ or $x^2+7x+\frac{49}{4}=\frac{9}{4}$ or $x+\frac{7}{2}=\pm\frac{3}{2}$ or $x=-2$ or -5.

20. $x^2-x+\frac{1}{4}=\frac{9}{4}$ or $x-\frac{1}{2}=\pm\frac{3}{2}$ or $x=2$ or -1.

21. $x^2-\frac{3}{2}x=1$ or $x^2-\frac{3}{2}x+\frac{9}{16}=\frac{25}{16}$ or $x-\frac{3}{4}=\pm\frac{5}{4}$, $x=2$ or $-\frac{1}{2}$.

22. $x^2+\frac{1}{2}x=\frac{1081}{2}$ or $x^2+\frac{1}{2}x+\frac{1}{16}=\frac{8649}{16}$ or $x-\frac{1}{4}=\pm\frac{93}{4}$.

23. $x^2+\frac{5}{6}x=1$ or $x^2+\frac{5}{6}x+\frac{25}{144}=\frac{169}{144}$ or $x+\frac{5}{12}=\pm\frac{13}{12}$ or $x=\frac{2}{3}$ or $-\frac{3}{2}$.

24. If $x^2+x=1\frac{1}{9}$, $x=\dfrac{2}{3}$ or $-\dfrac{5}{3}$, then $x+\dfrac{1}{x}=2\frac{1}{6}$ or $-2\frac{4}{15}$.

Exercise 120 — Page 248

1. $x^2-4x+4=5$ or $x=2\pm\sqrt{5}=2\pm 2.236=4.236$ or $-.236$.

2. $x^2-10x+25=8$ or $x=5\pm\sqrt{8}=5\pm 2.828=7.828$ or 2.172.

3. $x^2+2x+1=7$ or $x=-1\pm\sqrt{7}=-1\pm 2.646=1.646$ or -3.646.

4. $x^2+8x+16=35$ or $x=-4\pm\sqrt{35}=-4\pm 5.916=1.916$ or -9.916.

5. $x^2 + 3x + \frac{9}{4} = 3$ or $x = -\frac{3}{2} \pm \sqrt{3} = -\frac{3}{2} \pm 1.732 = .232$ or -3.232.

6. $x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{41}{16}$ or $x = -\frac{3}{4} \pm \frac{1}{4}\sqrt{41} = -\frac{3}{4} \pm \frac{1}{4} \times 6.403 = .851$ or -2.351.

13. If they are x and $11 - x$, then $x(11 - x) = 30$ or $x^2 - 11x + 30 = 0$ or $(x - 5)(x - 6) = 0$. $\therefore x = 5$ or 6 and $11 - x = 6$ or 5. The numbers are 5 and 6.

14. If they are x and $x + 1$, then $x^2 + (x + 1)^2 = 85$ or $x^2 + x - 42 = 0$ or $x = 6$ or -7. $\therefore x + 1 = 7$ or -6. The numbers are 6, 7 or -7, -6. Usually arithmetical numbers are meant, if so they are 6 and 7.

15. If they are x and $x + 13$, then $x^2 + 13x = 300$ and $x = 12$ or -15. If $x = 12$, $x + 13 = 25$ and the sides are 12, 25.

16. $(x + x + 1)^2 = x^2 + (x + 1)^2 + 220$ or $x^2 + x - 110 = 0$, $x = 10$ or -11.

17. If there were x yd., then cost per yd. $= \frac{5400}{x}$ cents. $\therefore \frac{5400}{x} = x + 30$ or $x^2 + 30x = 5400$ or $x^2 + 30x + 225 = 5625$ or $x + 15 = 75$ or $x = 60$.

18. If length is x rods, width is $x - 18$ rods, then $x(x - 18) = 1440$, $x = 48$.

19. If they are $x - 1$, x, $x + 1$, then $(x + 1)^2 = (x - 1)^2 + x^2$ or $x^2 - 4x = 0$, $x = 4$.

20. If the numbers are x and $x + 4$, then $x^2 + (x + 4)^2 = 730$, $x = 17$.

21. If it is x rods, then $(12 + x)(5 + x) = 120$ or $x^2 - 17x - 60 = 0$, $x = 3$.

22. $(x + 2)(x + 5) = 5(x + 1)(x - 1)$ or $4x^2 - 7x - 15 = 0$ or $x = 3$.

23. If it is x, then $(18 + 2x)(12 + 2x) = 432$ or $x = 3$.

24. $3\frac{1}{7}(r - 3)^2 = \frac{4}{9} \times 3\frac{1}{7} r^2$ or $r^2 - 6r + 9 = \frac{4}{9} r^2$ or $r = 9$.

25. If they are x, $x - 10$, $x - 5$, then $x^2 = (x - 10)^2 + (x - 5)^2$ or $x = 25$.

26. If it is \$$x$, then $\frac{90}{x} - \frac{90}{x + 1\frac{1}{2}} = 3$ or $2x^2 + 3x - 90 = 0$ or $x = 6$.

27. If he bought x, then $\frac{180}{x - 5} - \frac{200}{x} = 2$ or $x^2 + 5x - 500 = 0$ or $x = 20$.

28. If the tens is x and the units $9 - x$, then $x^2 + (9 - x)^2 = \frac{5}{7}(10x + 9 - x)$ or $14x^2 - 171x + 522 = 0$ or $(x - 6)(14x - 87) = 0$ or $x = 6$ and the no. $= 63$.

29. If it was x, then $\frac{400}{x - 2} - \frac{400}{x} = 10$ or $x^2 - 2x - 80 = 0$ or $x = 10$.

30. The diagonal is 10. If x is added, then $(8 + x)^2 + 6^2 = 12^2$ or $x^2 + 16x = 44$ or $x = -8 + \sqrt{108} = -8 + 6\sqrt{3} = 2.393$.

31. (1) If $OD = x$, then $16 \times 3 = 8x$ or $x = 6$.
(2) $40 = x(13 - x)$ or $x^2 - 13x + 40 = 0$, then $x = 5$ or 8.

33. If the cost was \$$x$, then $\frac{x}{100}$ of $x = 56 - x$ or $x = 40$.

34. If it is $\frac{x}{x + 3}$, then $\frac{x + 4}{x + 7} = \frac{6}{5} \times \frac{x}{x + 3}$ or $x^2 + 7x - 60 = 0$ or $x = 5$.

35. If the side is x in., then each side of the base is $(x-6)$ in. and the height is 3 in. $\therefore 3(x-6)^2 = 432$ or $x = 18$.

36. If A takes x days then B takes $x+12$, then $\frac{1}{x}+\frac{1}{x+12}=\frac{5}{72}$ or $x=24$.

Exercise 121 — Page 250

2. $x^3+2x^2-5x-6=x^3+2x^2-31x+28$. A simple equation, $x=1\frac{4}{13}$.

3. $x^2+x=4$ or $x^2+x+\frac{1}{4}=\frac{17}{4}$ or $x=-\frac{1}{2}\pm\frac{1}{2}\sqrt{17}=1.561$.

4. $3x^2-26x+40=0$, $x=2$ or $6\frac{2}{3}$; $x^2-17x+60=0$, $x=5$ or 12.

5. $(xy-7)(xy+1)=0$, $xy=7$ or -1.

6. The roots of $x^2=16$ are ± 4, so they are not equivalent.

7. $x^2-7y+49=39$ or $x^2-7y+10=0$, $x=5$ or 2.

8. If they are x and $14-x$, then $x^2+(14-x)^2=2x(14-x)+4$, $x=6$ or 8.

9. The conclusion is that $x-3=0$ or $x-2=7$.

10. $6440=16\cdot 1\,t^2$ or $t^2=400$, $t=20$.

11. $x^2-\frac{4}{3}x=\frac{1}{3}$ or $x^2-\frac{4}{3}x+\frac{4}{9}=\frac{7}{9}$ or $x=\frac{2}{3}\pm\frac{1}{3}\sqrt{7}$.

12. $10x=x^2+24$ or $x^2-10x+24=0$. Since x is either 6 or 4 it does not determine the number definitely.

13. $5x^2-11x-12=0$ or $(x-3)(5x+4)=0$, $x=3$ or $-\frac{4}{5}$.

14. If they are $x-1$ and $x+1$, $x^2-1=399$, $x=20$.

15. $(2x+3)^2-2(2x+3)-35=0$ or $(2x+3-7)(2x+3+5)=0$, $x=2$ or -4.

16. If they are x and x^2, then $x^2+x=12$ or $x=3$. The no. $=39$.

17. $x^2-11x-60=0$, $x=15$ or -4; $5x^2-8x+3=0$, $x=1$ or $\frac{3}{5}$.

18. If the rate is x mi. per hour, $\frac{315}{x}-\frac{315}{x+10}=2$, $x=35$.

19. $6x^2-41x+63-5x^2+42x-72=2x^2-7x+6$, $x=5$ or 3.

20. $x^2-x=1$ or $x^2-x+\frac{1}{4}=\frac{5}{4}$ or $x=\frac{1}{2}\pm\frac{1}{2}\sqrt{5}=1.618$.

21. $4x^2-3x(x+3)+(x+3)^2=14$ or $2x^2-3x-5=0$, $x=-1$ or $2\frac{1}{2}$.

22. If the length is x, the width is $17-x$, $x^2+(17-x)^2=169$, $x=12$ or 5.

23. $x=6$ or -2. Two numbers differ by 4 and the sum of their squares is 40; find the numbers.

24. If x is the shorter part, the other is $20-x$, then $x(20-x)=x^2+48$. $x=4$ or 6 and $20-x=16$ or 14. The parts are 4, 16 or 6, 14.

25. $x^3+(3-x)^3=9$ or $x^3+27-27x+9x^2-x^3=9$, $x=2$ or 1.

26. If the sides are $5x$ and $12x$, $(5x)^2 + (12x)^2 = 39^2$, $x = 3$.

27. Dividing by $x - 5$ we get $x^2 - 2x - 4 = 0$ or $x = 1 \pm \sqrt{5}$.

28. If the price is x cents per egg a dollar will buy $\frac{100}{x}$ eggs. If the price is $x + \frac{1}{3}$ cents each it would buy $\frac{100}{x + \frac{1}{3}}$ eggs. Then $\frac{100}{x} - \frac{100}{x + \frac{1}{3}} = 10$ or $x = 1\frac{2}{3}$. Price per doz. is 20 cents.

29. Let the sides be x and $x + 30$. Side of square field is $x + 15$. Then $(x + 15)^2 = \frac{25}{24}x(x + 30)$ or $x^2 + 30x - 5400 = 0$, $x = 60$.

30. $8x^2 - 4x = 11$ or $x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{23}{16}$ or $x = \frac{1}{4} \pm \frac{1}{4}\sqrt{23}$.

31. If there were x, then $\frac{20}{x} + \frac{1}{4} = \frac{20}{x - 4}$ or $x^2 - 4x - 320 = 0$, $x = 20$.

32. If the rate is x mi. per hour, then $\frac{36}{x} - \frac{36}{x + 1} = 3$ or $x = 3$.

33. If $\frac{1}{2}n(n - 3) = 20$, $n^2 - 3n - 40 = 0$ or $n = 8$.

34. Take the square root, then $a(x - a) = b(x + a)$ or $a(x - a) = -b(x + a)$.

35. If A's time is x then B's is $x + 10$, then $\frac{1}{x} + \frac{1}{x + 10} = \frac{1}{12}$, $x = 20$.

36. $8x^6 - 65x^3 + 8 = 0$ or $(8x^3 - 1)(x^3 - 8) = 0$, $x^3 = \frac{1}{8}$ or 8, $x = \frac{1}{2}$ or 2.

37. If the sides are $3x$ and $2x$ rods, then $6x^2 = 5.4 \times 160$, $x = 12$. The sides are 24, 36 rods. To contain 6 acres, the length must be $6 \times 160 \div 24 = 40$ rods or 4 rods more.

Exercise 122 — Page 255

1. $\frac{2}{3}$. **2.** $\frac{1}{2}$. **3.** $\frac{5}{7}$. **4.** $\frac{3}{2}$. **5.** $\frac{1}{3}$. **6.** $\frac{1}{4}$. **7.** $\frac{2}{9}$. **8.** 4.

9. $\frac{9}{13}$. **10.** 3. **11.** $\frac{b}{2a}$. **12.** $\frac{1}{a - b}$. **13.** $\frac{1}{a^2 + ab + b^2}$.

14. 1. **15.** $\frac{x - 1}{x + 1}$. **16.** $x - 3$. **17.** $\frac{x + y}{x^2 - xy + y^2}$.

18. 1 inch = 2.54 cm. $\therefore$ 1 in. : 1 cm. = 2.54 : 1.

1 meter = 100 cm. $= \frac{1200}{30.48}$ in. $= \frac{1200}{30.48 \times 36}$ yd. = 1.0936 yd.

20. 1 franc = \$.192, $\therefore$ 1 franc : \$1 = .192 : 1.

25¢ $= \frac{625}{480}$ francs = 1.302 francs.

21. 1000 meters = 39370 in. $= \frac{39370}{63360}$ mi. $= \frac{3937}{6336}$ mi. **22.** See Art. 176.

24. When added it is increased, and decreased when subtracted.

26. $2x + 3x + 4x = 360$, $x = 40$. The parts = 80, 120, 160.

27. If the parts are $2x$ and $3x$, then $2x + 3x = 165$.

If the parts are $3x$ and $7x$, then $3x + 7x = 510$.

If the parts are $1\frac{1}{2}x$ and $2\frac{1}{2}x$, then $1\frac{1}{2}x + 2\frac{1}{2}x = 36$.

28. Let the sum be $\$x$. In the first case the smaller part is $\frac{1}{3}$ of the sum and in the second $\frac{2}{9}$, then $\frac{1}{3}x - \frac{2}{9}x = 20$.

29. If $\dfrac{5+x}{8+x} = \dfrac{4}{5}$, $x = 7$.

30. If $\dfrac{7-x}{10-x} = \dfrac{13}{19}$, $x = \dfrac{1}{2}$.

31. If $\dfrac{a+x}{b+x} = \dfrac{c}{d}$, $x = \dfrac{ad-bc}{c-d}$. When $c = d$ it is impossible unless $a = b$, then x may have any value.

32. When reduced to a common denominator the numerators are $1 + 6a + 8a^2$ and $1 + 6a + 9a^2$ and therefore the second is the greater.

33. 30 miles per hour = 44 ft. per second. The ratio = 4 : 5.

34. If the parts are bx, cx, then $bx + cx = a$ or $x = \dfrac{a}{b+c}$.

35. Let their incomes be $3x$ and $4x$, and their expenses $5y$ and $6y$. Then $3x = 5y$ or $x = \frac{5}{3}y$. B saves $\dfrac{4x - 6y}{4x}$ of his income. This fraction $= \dfrac{6\frac{2}{3}y - 6y}{6\frac{2}{3}y} = \dfrac{1}{10}$ or 10 %.

36. If A gets $3x$, B gets $4x$ and C gets $5\frac{3}{5}x$. $\therefore 12\frac{3}{5}x = 315$.

37. In the first division the shorter part is $\frac{5}{12}$ of the line and in the second $\frac{3}{8}$, then $(\frac{5}{12} - \frac{3}{8})$ of the line = 1 in.

38. If they are $3x$ and $5x$, then $\dfrac{3x+10}{5x-10} = \dfrac{5}{3}$, $x = 5$.

39. In 1 sec. the first goes $\dfrac{m}{a}$ ft. and the second goes $\dfrac{n}{20b}$ ft. The ratio $= \dfrac{m}{a} : \dfrac{n}{20b} = 20bm : an$.

Exercise 123 — Page 258

1. 6. **2.** 21. **3.** $1\frac{1}{5}$. **4.** $7\frac{1}{2}$. **5.** -6. **6.** -10. **7.** $\dfrac{bc}{a}$.

8. $\dfrac{ac}{b}$. **9.** ± 8. **10.** 28. **11.** 6. **12.** $3\frac{1}{2}$. **13.** $3\frac{1}{2}$. **14.** $\frac{1}{4}$.

15. $\frac{1}{2}$. **16.** 8. **17.** $-1\frac{1}{2}$. **18.** $-\frac{3}{11}$. **19.** ± 2. **20.** $\pm 1\frac{1}{2}$.

21. 3, 5. **22.** All three equations are equivalent to $ad = bc$.

23. If it is x in each case $2x = 54$, $5x = 70$, $\frac{1}{2}x = \frac{1}{12}$, $ax = bc$, $ax = 6bc$.

24. $(a-b)x = (a^2 - b^2)(a+b)^2$ or $x = (a+b)^3$; $(a^2 - 3a + 2)x = (a^2 - 5a + 6)(a^2 - 5a + 4)$ or $x = (a-3)(a-4)$.

25. If $(2+x)(25+x) = (4+x)(17+x)$, then $x = 3$.

26. $(a+x)(d+x) = (b+x)(c+x)$ or $x = \dfrac{bc - ad}{a+d-b-c}$. If $bc = ad$ then $x = 0$ and a, b, c, d are proportionals.

27. $4(a+3)=3(a+15)$ or $a=33$.

28. If their ages are $4x$ and $5x$, then $\frac{4x-5}{5x-5}=\frac{3}{4}$, or $x=5$.

29. If the side is x, then $10 : x=\sqrt{3} : 2$ or $x=\frac{20}{\sqrt{3}}=\frac{20\sqrt{3}}{3}$.

30. If $AE=x$ then $EC=15-x$ so that $14 : 6=x : 15-x$ or $x=10\frac{1}{2}$.

31. If $DE=x$ then $x : 10=6 : 8$ or $x=7\frac{1}{2}$. If $AE=x$, $x : 9=6 : 8$.

32. If the area $=x$ then $735 : x=35^2 : 20^2$ or $x=240$.

33. $AC=10\sqrt{2}$. If $FC=x$ then $10 : 7=10\sqrt{2} : x$ or $x=7\sqrt{2}$.

34. If the bases are $3x$, $4x$ and the heights $8y$, $9y$ the ratio of their areas $=24xy : 36xy=2 : 3$.

35. $4x=8y$ or $\frac{x}{y}=2$; $x(a-c)=y(d-b)$ or $\frac{x}{y}=\frac{d-b}{a-c}$; $x(m-n)=y(m+n)$ or $\frac{x}{y}=\frac{m+n}{m-n}$; $px=-qy$ or $\frac{x}{y}=-\frac{q}{p}$.

36. $(3x-2y)(2x-3y)=0$, $\frac{x}{y}=\frac{2}{3}$, $\frac{3}{2}$; $(x-5y)(x+y)=0$, $\frac{x}{y}=5$, -1.

37. Eliminate c and $3a-5b=0$ or $\frac{a}{b}=\frac{5}{3}$. Eliminate b and $8a+5c=0$ or $\frac{a}{c}=\frac{5}{-8}$. Then $\frac{a}{5}=\frac{b}{3}=\frac{c}{-8}$.

38. If $DE=x$ then $3 : x=12 : 16$ or $x=4$. If $AE=y$ then since $AC=20$, $3 : y=12 : 20$ or $y=5$.

39. If the length is x, then $10 : 17\frac{1}{2}=84 : x$ or $x=147$.

40. Divide by y^2, then $3\left(\frac{x}{y}\right)^2-10\left(\frac{x}{y}\right)+3=0$ or $\frac{x}{y}=3$ or $\frac{1}{3}$.

41. If x is the tens digit and y the units digit then $10x+y : 10y+x=7 : 4$ or $x=2y$ and $10x+y+10y+x=66$ or $x+y=6$. Solving $x=4$, $y=2$ and the number is 42.

42. Let the length $=6x$, then the width $=5x$ and the height $4x$. Then $30x^2=187\frac{1}{2}$ or $x=2\frac{1}{2}$. The dimensions are 15, $12\frac{1}{2}$, 10.

43. Let the earnings be x, y, z then $4x+3y=16z$ and $6x+5z=10y$. Eliminate x and $y=2z$. Eliminate y and $x=2\frac{1}{2}z$. Then $x : y : z=5 : 4 : 2$.

44. $15ab+10b^2=18a^2-9ab$ or $18a^2-24ab-10b^2=0$ or $(3a-5b)(3a+b)=0$, $\frac{a}{b}=\frac{5}{3}$ or $-\frac{1}{3}$.

45. (1) If $BD=x$, $DC=12-x$, then $10 : 8=x : 12-x$ or $x=6\frac{2}{3}$.

(2) $c : b=x : a-x$, then $ac-cx=bx$ or $x=\frac{ac}{b+c}$.

46. If the sides are a and b, the diagonal $= \sqrt{a^2+b^2}$. Then $ab : a^2+b^2 = 6 : 13$ or $6a^2 - 13ab + 6b^2 = 0$ or $\frac{a}{b} = \frac{2}{3}$ or $\frac{3}{2}$.

47. Let its sides be $7x$, $10x$, $12x$, then $7x + 10x + 12x = 72\frac{1}{2}$ or $x = 2\frac{1}{2}$.

48. $7(x+y) = 5(2x - 3y)$ or $3x = 22y$; $9(x+y) = 5(x + 2y + 5z)$ or $4x - y - 25z = 0$. Eliminate x and $17y = 15z$. Then $x : y : z = 110 : 15 : 17$.

Exercise 124 — Page 262

1. ± 8, $\pm 4a$, $\pm 6a^2b^2$, $\pm(a^2 - b^2)$. **2.** 8, 300, $20ab^2$, $\frac{x-y}{x+y}$.

3. If they are x and $34 - x$, then $x(34 - x) = 225$ or $x^2 - 34x + 225 = 0$. Then $x = 9$ or 25 and the numbers are 9 and 25.

4. If the first is x, then the third is $51 - x$, therefore $x(51 - x) = 144$ or $x^2 - 51x + 144 = 0$, $x = 48$ or 3.

5. If $(3 + x)(12 + x) = (7 + x)^2$, then $x = 13$.

6. (1) If $AD = x$, then $x^2 = 36$, $x = 6$. (2) If $DC = x$, then $64 = 5(5 + x)$ or $x = 7\frac{4}{5}$. (3) If $DC = x$, then $144 = 13x$ or $x = 11\frac{1}{13}$, $BD = 1\frac{12}{13}$. $AB^2 = BD \cdot BC = 13 \times 1\frac{12}{13} = 25$ or $AB = 5$. $AD^2 = BD.DC = 1\frac{12}{13} \times 11\frac{1}{13}$ or $AD = 4\frac{8}{13}$. (4) $BC^2 = AB^2 + AC^2$ or $BC = 5$. If $BD = x$, then $9 = 5x$ or $x = 1\frac{4}{5}$ and $DC = 3\frac{1}{5}$. $AD^2 = 1\frac{4}{5} \times 3\frac{1}{5}$ or $AD = 2\frac{2}{5}$.

7. (1) In the diagram make $BD = 2$ and $DC = 3$, then $AD = \sqrt{6}$.
(2) Make $BD = 3$ in., $DC = 4$ in., then $AD^2 = 12$ sq. in.

8. If they are x and y, then $xy = 16$ and $32x = y^2$. Divide the second equation by the first and $y^3 = 512$ or $y = 8$.

9. If the shortest is x, the longest is $4x$ and the other is $21 - 5x$. Then $(21 - 5x)^2 = 4x^2$ or $x^2 - 10x + 21 = 0$ or $x = 3$ or 7. If $x = 3$ the parts are 3, 6, 12. $x = 7$ gives an inadmissible solution.

Exercise 125 — Page 264

1. Let $a = bk$, $c = dk$, then $\frac{a}{2a+3b} = \frac{bk}{2bk+3b} = \frac{k}{2k+3}$.

$\frac{c}{2c+3d} = \frac{dk}{2dk+3d} = \frac{k}{2k+3}$. $\therefore$ the fractions are equal.

2. $\frac{ma+nb}{ma-nb} = \frac{mbk+nb}{mbk-nb} = \frac{mk+n}{mk-n}$; $\frac{mc+nd}{mc-nd} = \frac{mdk+nd}{mdk-nd} = \frac{mk+n}{mk-n}$.

3. $a^2bd + b^2c + bc = b^3k^2d + b^2dk + bdk$.
$ab^2c + abd + ad = b^3k^2d + b^2dk + bdk$.

5. Since $x = \frac{2}{3}y$, the numerator $= (\frac{4}{3}y + 3y)(2y + 2y) = \frac{52}{3}y^2$. The denominator $= (\frac{10}{3}y - 3y)(2y - 5y) = -y^2$. The fraction $= -17\frac{1}{3}$.

6. $x = 3y$, $a = \frac{2}{5}b$, then $ax = \frac{6}{5}by$. Now substitute for ax.

7. Let $x = ak, y = bk, z = ck$, then $\dfrac{ak + bk + ck}{a + b + c} = k$.

8. $x = (b - c)k$, $y = (c - a)k$, $z = (a - b)k$, then $x(b + c) + y(c + a) + z(a + b) = (b^2 - c^2)k + (c^2 - a^2)k + (a^2 - b^2)k = 0$.

9. $\dfrac{a^3c + ac^3}{b^3d + bd^3} = \dfrac{b^3dk^4 + bd^3k^4}{b^3d + bd^3} = k^4$; $\dfrac{a^2c^2}{b^2d^2} = \dfrac{b^2d^2k^4}{b^2d^2} = k^4$.

10. $a = k(x + y)$, $b = k(y - z)$, $c = k(z + x)$, then $b + c = k(y - z) + k(z + x) = k(x + y) = a$.

11. Since $\dfrac{a}{b} = \dfrac{b}{c}$, then $\dfrac{a^2}{b^2} = \dfrac{b}{c} \times \dfrac{a}{b} = \dfrac{a}{c}$.

12. Since $\dfrac{a - b}{a + b} = \dfrac{c - d}{c + d}$, then $\dfrac{2a}{2b} = \dfrac{2c}{2d}$ or $\dfrac{a}{b} = \dfrac{c}{d}$. (See Art. 185.)

13. See Art. 185, Ex. 2. **14.** See Art. 185, Ex. 1.

15. If $x + y : x - y = 7 : 4$, then $4(x + y) = 7(x - y)$ or $3x = 11y$.

17. Using Art. 185, $\dfrac{ax + c}{b} = \dfrac{bx + c}{a}$ or $x = \dfrac{bc - ac}{a^2 - b^2} = \dfrac{-c}{a + b}$.

18. $\dfrac{a}{b} = \dfrac{3}{5}$, $\dfrac{b}{c} = \dfrac{7}{9}$, $\dfrac{c}{d} = \dfrac{15}{16}$. Multiply together and $\dfrac{a}{d} = \dfrac{7}{16}$.

20. $\dfrac{x + y}{4} = \dfrac{x - y}{2} = \dfrac{xy}{9}$, then $2(x + y) = 4(x - y)$ or $x = 3y$. Also $9(x - y) = 2xy$ or $9(3y - y) = 6y^2$, then $y = 3$, $x = 9$.

21. Substitute $b = a + 1$, $c = a + 2$, and $a = 19$, $b = 20$, $c = 21$.

22. If the length $= 3x$ and width $= 2x$, then $\dfrac{(3x + 2)(2x + 2)}{6x^2} = \dfrac{35}{27}$, or $8x^2 - 45x - 18 = 0$, or $x = 6$. The length $= 18$, width $= 12$.

23. $\dfrac{a}{a + b} = \dfrac{bk}{bk + b} = \dfrac{k}{k + 1}$; $\dfrac{a + c}{a + b + c + d} = \dfrac{bk + dk}{bk + b + dk + d} = \dfrac{k}{k + 1}$.

24. Cross multiply and $120ac + 10ad + 12bc + bd = 120ac + 10bc + 12ad + bd$, or $2bc = 2ad$, or $\dfrac{a}{b} = \dfrac{c}{d}$.

25. Since $b^2 = ac$, $\dfrac{a^2 + ab}{ac} = \dfrac{a + b}{c}$; $\dfrac{ac + bc}{c^2} = \dfrac{a + b}{c}$.

Exercise 126 — Page 266

1. $\dfrac{5}{6}$. **2.** $\dfrac{x+y}{x-y}$. **3.** $\dfrac{a^2-ab+b^2}{a+b}$. **4.** $\dfrac{x-1}{x}$. **5.** a^2+ab+b^2.

6. $\dfrac{a}{c}$. **7.** $\dfrac{x+4}{x-2}$. **8.** $\dfrac{a^2+a+1}{a}$.

9. If they are $3x$, $4x$, $11x$, then $18x = 144$. The parts $= 24$, 32, 88.

10. If $4 + x : 7 + x = 6 : 7$, then $x = 14$.

11. $\frac{3}{2} = \frac{9}{6}$ or $\frac{3}{9} = \frac{2}{6}$; $\frac{2}{3} = \frac{x}{5}$ or $\frac{2}{x} = \frac{3}{5}$; $\frac{a}{c} = \frac{d}{b}$ or $\frac{a}{d} = \frac{c}{b}$; $\frac{a+b}{3} = \frac{4}{a-b}$ or $\frac{a+b}{4} = \frac{3}{a-b}$; $\frac{a-2}{a+4} = \frac{a+1}{a-3}$ or $\frac{a-2}{a+1} = \frac{a+4}{a-3}$.

12. 28. **13.** 75, a^4, $(x-y)^2$, $(a-b)^2$.

14. If they are $9x$, $5x$, then $81x^2 - 25x^2 = 504$, $x = \pm 3$.

15. If they are $5x$, $8x$, then $\frac{5x+8}{8x-2} = \frac{14}{15}$, $x = 4$.

16. If they are $6x$, $5x$, then $\frac{6x+5x}{36x^2 - 25x^2} = \frac{1}{3}$, $x = 3$.

17. If $\frac{a-x}{b-x} = \frac{a^2}{b^2}$, then $x = \frac{a^2b - ab^2}{a^2 - b^2} = \frac{ab}{a+b}$.

22. $63(x-1)(x^2+x+1) = 62(x+1)(x^2-x+1)$ or $x^3 = 125$.

23. Eliminate z and $4x = 3y$. Eliminate y and $5x = 3z$. Then $\frac{x}{3} = \frac{y}{4}$ and $\frac{x}{3} = \frac{z}{5}$, or $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

24. $x = \frac{2}{3}y$, then $\frac{6x - 2y}{3x + 11y} = \frac{4y - 2y}{2y + 11y} = \frac{2}{13}$.

25. If it is a, then $a^2 = \frac{x^2y^2 - 1}{y^2} \cdot \frac{x^2y^2 - 1}{x^2}$, $a = \frac{x^2y^2 - 1}{xy}$.

26. $a = \frac{3}{2}b$, then $\frac{ax + by}{bx + ay} = \frac{9}{11}$, or $\frac{\frac{3}{2}bx + by}{bx + \frac{3}{2}by} = \frac{9}{11}$, or $11(\frac{3}{2}x + y) = 9(x + \frac{3}{2}y)$, $y = 3x$.

28. See Ex. 20, page 265. **30.** See Ex. 8, page 264. **31.** Ex. 7, page 264.

32. Eliminate z and $x = 2y$, eliminate x and $z = -y$. Substitute in $x^2 + y^2 + z^2 = 150$ and $4y^2 + y^2 + y^2 = 150$, $y = \pm 5$.

33. If the shortest side $= 5x$, the hyp. $= 13x$, and then the other side $= 12x$; $5x + 12x + 13x = 120$, $x = 4$, sides $= 20$, 48, 52.

34. Let them be $4x$, $3x$, $2x$, then the area of the four walls $= 2x(8x + 6x) = 28x^2$. The increased area $= (2x+2)(8x + 4 + 6x + 4) = (2x+2)(14x+8)$. $\therefore 280x^2 = 7(2x+2)(14x+8)$ or $3x^2 - 11x - 4 = 0$ or $x = 4$.

35. $\frac{a^3 + c^3 + e^3}{b^3 + d^3 + f^3} = \frac{b^3k^3 + d^3k^3 + f^3k^3}{b^3 + d^3 + f^3} = k^3$; $\frac{ace}{bdf} = \frac{bdfk^3}{bdf} = k^3$.

36. If the segments are x, $4\frac{2}{3} - x$, then $\frac{6}{8} = \frac{x}{4\frac{2}{3} - x}$. (Ex. 45, p. 260.)

37. Cross multiply and $x^2 + y^2 - z^2 + 2xy = z^2 - x^2 - y^2 + 2xy$.

38. If their incomes are $2x$, $3x$ and expenses $6y$, $7y$, then A saves $2x-6y$, and B saves $3x-7y$. If $2x-6y=\frac{1}{2}x$, $x=4y$. Then $3x-7y=12y-7y=5y$. Therefore B saves $\frac{5y}{3x}$ or $\frac{5y}{12y}$ or $\frac{5}{12}$, or $41\frac{2}{3}\%$ of his income.

39. If $x^3-8x^2y+19xy^2-12y^3=0$, $(x-3y)(x-4y)(x-y)=0$ or $\frac{x}{y}=3, 4, 1$.

Exercise 127 — Page 270

3. $x^2-6mx=-3m^2$, or $x^2-6mx+9m^2=6m^2$, or $x=3m\pm m\sqrt{6}$.

4. $x^2+4px+4p^2=5p^2$, or $x+2p=\pm p\sqrt{5}$, or $x=-2p\pm p\sqrt{5}$.

7. $x^2+2x=\frac{b}{a}$, or $x^2+2x+1=\frac{b}{a}+1$, or $x=-1\pm\sqrt{\frac{b}{a}+1}$.

8. $x^2+\frac{2b}{a}x=-\frac{c}{a}$, or $x^2+\frac{2b}{a}x+\frac{b^2}{a^2}=-\frac{c}{a}+\frac{b^2}{a^2}$, or $x=-\frac{b}{a}\pm\frac{1}{a}\sqrt{b^2-ac}$.

9. $x^2-\frac{b}{a}x+\frac{b^2}{4a^2}=+\frac{c}{a}+\frac{b^2}{4a^2}$, or $\left(x-\frac{b}{2a}\right)^2=\frac{b^2+4ac}{4a^2}$, or $x=\frac{b}{2a}\pm\frac{\sqrt{b^2+4ac}}{2a}$.

Exercise 128 — Page 271

1. $a=3$, $b=-5$, $c=2$, then $x=\frac{5\pm\sqrt{25-24}}{6}=\frac{5\pm 1}{6}=1$, or $\frac{2}{3}$.

2. $a=24$, $b=-46$, $c=21$, then $x=\frac{46\pm\sqrt{2116-2016}}{48}=\frac{46\pm 10}{48}=\frac{3}{4}$, or $\frac{7}{6}$.

3. $a=575$, $b=-2$, $c=-1$, then $x=\frac{2\pm\sqrt{4+2300}}{1150}=\frac{2\pm 48}{1150}=\frac{1}{23}$, or $-\frac{1}{25}$.

4. $a=2$, $b=-6$, $c=-1$, then $x=\frac{6\pm\sqrt{36+8}}{4}=\frac{6\pm 2\sqrt{11}}{4}=\frac{3\pm\sqrt{11}}{2}$.

5. $a=247$, $b=5$, $c=-12$, then $x=\frac{-5\pm\sqrt{25+11856}}{494}=\frac{-5\pm 109}{494}=\frac{4}{19}$ or $-\frac{3}{13}$.

6. $x=\frac{13\pm\sqrt{169-80}}{4}=\frac{13\pm\sqrt{89}}{4}$.

7. $x=\frac{-4\pm\sqrt{16+54756}}{782}=\frac{-4\pm 234}{782}$.

8. $x = \frac{10 \pm \sqrt{100 + 4800}}{2400} = \frac{10 \pm 70}{2400}$.

9. $x = \frac{2a - 3b \pm \sqrt{(2a - 3b)^2 + 24ab}}{2}$.

10. $x = \frac{25 \pm \sqrt{625 - 616}}{4} = \frac{25 \pm 3}{4}$. **11.** $x = \frac{1 \pm \sqrt{1 + 24}}{12} = \frac{1 \pm 5}{12}$.

12. $x = \frac{5 \pm \sqrt{25 + 7200}}{3600} = \frac{5 \pm 85}{3600}$.

13. $(3x - 4)(9x + 4) = 0$, $x = \frac{4}{3}$, or $-\frac{4}{9}$.

14. $(5x - 1)(3x + 2) = 0$, $x = \frac{1}{5}$, or $-\frac{2}{3}$.

15. $(4x - 3)(3x + 2) = 0$, $x = \frac{3}{4}$, or $-\frac{2}{3}$.

16. $(x - 4)(4x - 1) = 0$, $x = 4$, or $\frac{1}{4}$.

17. $(20x - 1)(23x + 1) = 0$, $x = \frac{1}{20}$, or $-\frac{1}{23}$.

18. $(5 - x)(1 - 5x) = 0$, $x = 5$, or $\frac{1}{5}$. **19.** $x = \frac{9 \pm \sqrt{81 + 80}}{10} = \frac{9 \pm \sqrt{161}}{10}$.

20. $x = \frac{9 \pm \sqrt{81 - 24}}{6} = \frac{9 \pm \sqrt{57}}{6}$.

21. $(2x - 3)(2x + 1) = 0$, $x = \frac{3}{2}$, or $-\frac{1}{2}$.

22. $x = \frac{4 \pm \sqrt{1280}}{8} = \frac{4 \pm 16\sqrt{5}}{8}$.

23. $(2y - 1)(2y + 3) = 0$, $y = \frac{1}{2}$, or $-\frac{3}{2}$.

24. $(x - 3)(x + 6) = 0$, $x = 3$ or -6.

25. $(x - 1)(x + 3) = 0$, $x = 1$ or -3.

26. $(3y - 1)(3y + 2) = 0$, $y = \frac{1}{3}$ or $-\frac{2}{3}$.

27. $x = \frac{7 \pm \sqrt{49 + 48}}{24} = \frac{7 \pm \sqrt{97}}{24}$.

28. $x^2 - 11x + 28 = 0$, $x = 4$ or 7. **29.** $2x^2 - 5x + 2 = 0$, $x = 2$ or $\frac{1}{2}$.

30. $(ax - 1)(2x + 1) = 0$, $x = \frac{1}{a}$, or $-\frac{1}{2}$.

31. $(x - a)(acx - b) = 0$, $x = a$, or $\frac{b}{ac}$.

32. $2x^2 - 4x - a^2 + 2 = 0$, $x = \frac{4 \pm \sqrt{16 + 8(a^2 - 2)}}{4} = \frac{2 \pm a\sqrt{2}}{2}$.

33. $x^2 = 6$, $x = \pm\sqrt{6}$. **34.** $3x^2 - 4x - 15 = 0$, $x = 3$, or $-1\frac{2}{3}$.

35. $x^2 - 2x - 23 = 0$, $x = \frac{2 \pm \sqrt{96}}{2} = 1 \pm 2\sqrt{6}$.

36. $x^2 - 2x - 12 = -12$, $x = 0$, or 2.

37. $y^2 + 12y - 13 = 0$, $y = 1$ or -13.

38. $x^2 - 2x - 1 = 0$, $x = 1 \pm \sqrt{2}$.

39. The roots are $\frac{3}{2} + \frac{1}{2}\sqrt{89}$, $\frac{3}{2} - \frac{1}{2}\sqrt{89}$.

40. If it is x ft., then $x^2 = 48\,x$, or $x = 48$.

41. If the length is x rods, then $x(x - 16) = 5120$ or $x = 80$.

42. If they are x, $x + 2$, $x + 4$, then $3\,x + 6 = \frac{1}{6}\,x(x+2)$, or $x = 18$.

43. If x is the shorter, then $(10 - x)^2 = 10\,x$, or $x = 15 \pm 5\sqrt{5}$.

44. If they are x, $x - 3$, then $x^2 + (x - 3)^2 = 317$, or $x = 14$, or -11.

45. If the side is x in., then $(x + 5)(x + 12) = 2\,x^2$ or $x = 20$.

46. $3\,x^2 - 8\,x = 1$, $x = \frac{1}{3}(4 \pm \sqrt{19}) = 2.786$, or $-.120$.

47. $x^2 - 1809 = x - 3$, or $x^2 - x - 1806 = 0$, $x = 43$.

48. If x is the length, $100\frac{1}{2} - x =$ width, then $x(100\frac{1}{2} - x) = 2420$ or $2\,x^2 - 201\,x + 4840 = 0$ or $x = 60\frac{1}{2}$ or 40.

49. If 8 is a root, $64 - 40 + d = 0$ or $d = -24$. The other is -3.

50. If the rate is x mi. per hour, $\frac{480}{x} - \frac{480}{x + 10} = 4$, $x = 30$.

51. If the parts are x, $3 - x$, then $x^2 + (3 - x)^2 = 4\frac{5}{9}$, $x = 1\frac{1}{3}$ or $1\frac{2}{3}$.

52. If x is the no., then $\frac{595}{x - 2} - \frac{480}{x} = \frac{1}{2}$ or $x^2 - 232\,x - 1920 = 0$, $x = 240$.

53. In each case let $AC = x$, then $CB = 12 - x$. In (1) $x^2 = 2(12 - x)^2$ or $x = 24 - 12\sqrt{2}$. In (2) $x^2 = 24(12 - x)$ or $x = -12 + 12\sqrt{3}$. In (3) $3\,x^2 = 48(12 - x)$ or $x = 8$. In (4) $x^2 + 3(12 - x)^2 = 288$ or $x = 9 - 3\sqrt{5}$. In (5) $x^2 - (12 - x)^2 = 10$ or $x = 6\frac{5}{12}$. In (6) $x(24 - x) = 288$ or $x = 12 \pm \sqrt{-144}$ which is impossible because -144 has no real square root.

54. If x cents is the cost of each, $\frac{600}{x - 5} - \frac{600}{x} = 6$, or $x = 25$.

55. $4\,a^2x^2 + 4\,abx = -4\,ac$ or $4\,a^2x^2 + 4\,abx + b^2 = b^2 - 4\,ac$ or $2\,ax + b = \pm\sqrt{b^2 - 4\,ac}$ or $x = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2\,a}$.

56. Multiply by $3(4 - x^2)$, then $3(3 + 2\,x)(2 + x) - 3(2 - 3\,x)(2 - x) - 3(16\,x - x^2) = 4 - x^2$ or $x^2 - 3\,x + 2 = 0$ or $x = 1$ or 2. On verifying, 1 is found to be a root but 2 is not.

Exercise 129 — Page 275

1. $x^2 = 4$ or 1, $x = \pm 2$ or ± 1.

2. $x^2 = 4$ or 9, $x = \pm 2$ or ± 3.

3. $y^2 = 3$ or $\frac{4}{9}$, $y = \pm\sqrt{3}$ or $\pm\frac{2}{3}$.

4. $x^3 = 8$ or $\frac{1}{8}$, $x = 2$ or $\frac{1}{2}$.

5. $(x + 2)(x + 3)(x - 7)(x - 2) = 0$, $x = -2, -3, 7, 2$.

6. Let $\frac{x^2 + 16}{25} = y$, then $y + \frac{1}{y} = 2$ or $y = 1$. Then $x = \pm 3$.

7. Let $x^2-4x=y$, then $(y+5)(y+2)=-2$ or $y=-3$ or -4. If $x^2-4x=-3$, $x=3$ or 1. If $x^2-4x=-4$, $x=2$.

8. Let $x^2+x=y$, then $y+1=\dfrac{42}{y}$ or $y=6$ or -7. If $x^2+x=6$, $x=2$ or -3. If $x^2+x=-7$, $x=-\frac{1}{2}\pm\frac{1}{2}\sqrt{-27}$.

9. Let $x^2+x+1=y$, then $y=1$ or 3 and $x=0, -1, 1, -2$.

10. $x^2(x-4)-4(x-4)=0$ or $(x-4)(x^2-4)=0$, $x=4$ or ± 2.

11. Let $x+\dfrac{1}{x}=y$, then $y=\frac{5}{2}$ or $\frac{10}{3}$. Then $x=2, \frac{1}{2}, 3, \frac{1}{3}$.

12. Let $x+x^2=y$, then $y=12$ or -13. Then $x=3, -4, -\frac{1}{2}\pm\frac{1}{2}\sqrt{-51}$.

13. $(x^2+5x+4)(x^2+5x+6)=120$. Let $x^2+5x+4=y$, then $y=10$ or -12. If $y=10$, $x=1, -6$. If $y=-12$, $x=-\frac{5}{2}\pm\frac{1}{2}\sqrt{-39}$.

15. $x^3-8=0$ or $(x-2)(x^2+2x+4)=0$. Then $x=2$ or $x^2+2x+4=0$.

16. $(x-2)(x+2)(x^2+4)=0$ or $x=2, -2, \pm 2\sqrt{-1}$.

17. Divide by $x-3$ and solve the resulting equation.

18. $x-1$ is evidently a factor. Divide by $x-1$ and solve.

19. $x^3-3x^2+2x-24=0$. Divide by $x-4$ and solve.

20. $(x^3-27)(8x^3-1)=0$ or $(x-3)(x^2+3x+9)(2x-1)(4x^2+2x+1)=0$. Put each factor equal to 0 and solve the equations.

22. $\left(x+\dfrac{1}{x}\right)^2+\left(x+\dfrac{1}{x}\right)=6$. Let $x+\dfrac{1}{x}=y$, then $y=2$ or -3. Now solve $x+\dfrac{1}{x}=2$ and $x+\dfrac{1}{x}=-3$.

Exercise 130 — Page 276

1. (*a*) By factoring: $(x-3)(3x+5)=0$ or $x=3$ or $-\frac{5}{3}$.

(*b*) By completing the square: $x^2-\frac{4}{3}x=5$ or $x^2-\frac{4}{3}x+\frac{4}{9}=\frac{49}{9}$ or $x-\frac{2}{3}=\pm\frac{7}{3}$ or $x=\frac{2}{3}\pm\frac{7}{3}=3$ or $-\frac{5}{3}$.

(*c*) By formula: $a=3$, $b=-4$, $c=-15$; $x=\dfrac{4\pm\sqrt{196}}{6}=3$ or $-\frac{5}{3}$.

2. $\frac{1}{19}, -\frac{1}{17}$. **3.** 2, 10. **4.** 2.73, $-.73$. **5.** $15c$.

6. $\frac{19}{20}\pm\frac{1}{20}\sqrt{721}$. **7.** 40, 30, 50. **8.** 5, 7. **9.** 20, 25.

10. $\frac{1}{75}, -\frac{1}{85}$. **11.** 20, 25. **12.** $\frac{1}{10}, 1\frac{9}{10}$.

13. The sides are 12, 16. The diagonal $=20$. **14.** 100, -200.

15. $x^2-5x+6=a^2-5a+6$ or $(x-a)(x+a)-5(x-a)=0$ or $(x-a)(x+a-5)=0$. The quantity therefore is $5-a$.

16. $(x-1)(x-10)(x+9)=0$ or $x=1, 10, -9$.

17. If the rates are x and $x+5$, then $\frac{330}{x}-\frac{330}{x+5}=\frac{1}{2}$, $x=55$.

18. If $\frac{x^2+2}{x+1}=y$, then $y=\frac{3}{2}$ or $\frac{2}{3}$ and $x=1,\ \frac{1}{2},\ \frac{1}{3}\pm\frac{1}{3}\sqrt{-11}$.

19. If $\frac{x}{x^2+1}=y$, then $y=\frac{3}{10}$ or $\frac{10}{3}$ and $x=3,\ \frac{1}{3},\ \frac{3}{20}\pm\frac{1}{20}\sqrt{-391}$.

20. If the cost was \$$x$, then $\frac{x}{100}$ of $x=96-x$, $x=60$.

21. If $x^2-3x-5=y$, then $y=-1$ or -7 and $x=4,\ -1,\ 1,\ 2$.

22. If the parts are x and $25-x$, then $\frac{x}{25-x}+\frac{25-x}{x}=\frac{17}{4}$, $x=5$.

23. $(x+17)^2+(x-17)^2=2500$, then $x=31$. The area $=672$.

24. If $x^2+x=y$, then $y=6$ or 12 and $x=2,\ -3,\ 3,\ -4$.

25. $\{(m-n)x+(m+n)\}\{(m+n)x+(m-n)\}=0$, $x=\frac{m+n}{n-m},\ \frac{n-m}{n+m}$.

26. $(x^2+x-6)(x^2+x-2)=60$. Let $x^2+x=y$, then $y=12$ or -4.

27. $x^3-125=0$ or $(x-5)(x^2+5x+25)=0$. Then $x=5$ or $x^2+5x+25=0$.

28. It will be zero when $x=2$ or 6. It will be negative for all values of x between 2 and 6.

29. $\frac{x(a-b)}{(x-a)(x-b)}=\frac{a-b}{a+b}$ or $(x-a)(x-b)=x(a+b)$ or $x^2-2x(a+b)+ab=0$.

30. $acx^2-adx+bcx-bd=0$ or $(cx-d)(ax+b)=0$.

31. If the side is x, then $(x+10)(x+12)=3x^2$ or $x=15$.

32. $[x(a-c)-2b][x(a+c)+b]=0$ or $x=\frac{2b}{a-c}$ or $\frac{-b}{a+c}$.

33. $(x^2+6x+8)(x^2+9x+8)=0$. Then $x^2+6x+8=0$ or $x^2+9x+8=0$.

34. If x is the no., then $\frac{300}{x-2}-\frac{300}{x}=5$ or $x=12$.

35. $\frac{1}{x+a+b}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$ or $\frac{-a-b}{x(x+a+b)}=\frac{a+b}{ab}$ or $x^2+x(a+b)+ab=0$.

36. $\frac{ax-a^2+bx-b^2}{ab}=\frac{ax-a^2+bx-b^2}{(x-a)(x-b)}$. Then $ax-a^2+bx-b^2=0$, and $x=\frac{a^2+b^2}{a+b}$ or $(x-a)(x-b)=ab$ and $x=0$ or $a+b$.

37. If it is x hours after noon, then A has gone $4x$ and B $4(x-2)$ miles. Then $16x^2+16(x-2)^2=400$, or $25x^2-64x-336=0$, or $x=\frac{32\pm\sqrt{9424}}{25}=5.16$ hr. or 5 hr. 10 min.

38. $(ax - a^2 + 1)(ax - a^2 - 1) = 0$, or $x = a - \frac{1}{a}$, or $a + \frac{1}{a}$.

39. $(a+b)^2x^2 - c(a+b)x - 6c^2 = 0$ or $\{x(a+b) - 3c\}\{x(a+b) + 2c\} = 0$.

40. If the side of the field is x yd. then the length of the walk is $4x - 8$, then $2(4x - 8) = \frac{1}{10}x^2$, or $x^2 - 80x + 160 = 0$, or $x = 40 \pm \sqrt{1440} = 40 \pm 37.95 = 77.95$, since $x = 2.05$ is impossible.

42. If it is x, then $x^3 + x^2 = 9(x + 1)$. Divide by $x + 1$ and $x = 3$.

43. If $x^2 + x - 2 = y$, then $y = 3$ or 1.

44. If they are x in. from the corners, then $x^2 + (34 - x)^2 = 676$.

45. Divide by y^2, then $a\left(\frac{x}{y}\right)^2 + b\left(\frac{x}{y}\right) + c = 0$, $\frac{x}{y} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

46. If $x^2 + 10x = 1000$, $x = -5 \pm \sqrt{1025}$. The nearest positive integral value of x is 27.

Exercise 131 — Page 281

1. $x = 7 - y$, then $(7 - y)y = 12$, or $y^2 - 7y + 12 = 0$, $y = 3$ or 4, $x = 4$ or 3.

2. $x = 4 + y$, then $(4 + y)y = 60$, or $y^2 + 4y - 60 = 0$, $y = 6$, -10, $x = 10$, -6.

3. $x = 2y$, then $4y^2 - y^2 = 27$, $y = \pm 3$, $x = \pm 6$.

4. $x = 3 + y$, then $(3 + y)^2 + y^2 = 65$, or $y^2 + 3y - 28 = 0$, $y = 4$, -7, $x = 7$, -4.

5. Divide the second by the first, then $x + y = 10$, $x = 8$, $y = 2$.

6. $y = 9 - 2x$, then $x^2 - (9 - 2x)^2 = 15$, or $x^2 - 12x + 32 = 0$, $x = 4$, 8, $y = 1$, -7.

7. $x = 11 - 3y$, then $(11 - 3y)^2 + y = 27$, or $9y^2 - 65y + 94 = 0$. $y = 2$ or $\frac{47}{9}$. When $y = 2$, $x = 5$; when $y = \frac{47}{9}$, $x = -\frac{14}{3}$.

8. $x = \frac{12 - 3y}{2}$, then $\frac{(12 - 3y)^2}{4} + y^2 = 13$ or $13y^2 - 72y + 92 = 0$.

9. $x = \frac{2 + 4y}{3}$, then $\frac{(2 + 4y)^2}{3} + 2y^2 = 140$, or $11y^2 + 8y - 208 = 0$.

10. $x = 3 + y$, then $(3 + y)^2 + 3y(3 + y) + y^2 + 2(3 + y) = 37$, or $5y^2 + 17y - 22 = 0$.

11. $x = \frac{1}{2}(1 + 3y)$, then $\frac{3}{4}(1 + 3y)^2 - y(1 + 3y) + \frac{5}{2}(1 + 3y) - y = 17$ or $3y^2 + 8y - 11 = 0$.

12. $x = 2 + 3y$, then $(2 + 3y)^2 - y(2 + 3y) + 2y^2 = 6$, or $4y^2 + 5y - 1 = 0$, $y = \frac{-5 \pm \sqrt{41}}{8} = \frac{-5 \pm 6.403}{8} = .175$ or -1.425.

13. If they are x and y, then $x^2 + y^2 = 625$, $x + y = 31$; $x = 7$, $y = 24$.

14. If they are x and $x + 10$, then $(x - 8)^2 + (x + 2)^2 = 148$, $x = 10$.

15. If the sides are x and y, then $x-y=10$ and $x^2+y^2=2500$. Solving $x=40$, $y=30$, and the area $=1200$.

16. If the sides are x and y, then $x-y=4$ and $\frac{1}{2}xy=96$. Solving $x=16$, $y=12$, then the hyp. $=20$.

17. $x=\frac{2-5y}{3}$, then $\frac{(2-5y)^2}{3}-10y^2-\frac{y(2-5y)}{3}+28=0$. This is a simple equation from which $y=4$, then $x=-6$.

18. Let x be the tens' digit and y the units, then $(x+2)(y+2)=10x+y$ and $10y+x=13x$, or $x=\frac{5}{6}y$. Substitute for x in the first equation and $y=6$, $x=5$.

19. Let $x=$ side of the larger and $y=$ smaller, then $y=3x-10$ and $x^2+y^2=40$ or $x^2+(3x-10)^2=40$ or $x^2-6x+6=0$; $x=3\pm\sqrt{3}=4.732$ or 1.268 and $y=4.196$ or -6.196. The sides are 4.732 and 4.196.

20. From (1) $\frac{x}{y}=2$ or -7 or $x=2y$ or $-7y$. If $x=2y$, then $y-1=2y$ or $y=-1$, $x=-2$. If $x=-7y$, $y=\frac{1}{8}$, $x=-\frac{7}{8}$.

Exercise 132 — Page 282

1. From (1) $x=y$ or $-y$. If $x=y$, then $3y^2=36$, $y=\pm 2\sqrt{3}$ and $x=\pm 2\sqrt{3}$. If $x=-y$, then $y^2=36$, $y=\pm 6$ and $x=\mp 6$.

2. From (1) $x=3y$ or y. If $x=3y$, $10y^2=10$, $y=\pm 1$, $x=\pm 3$. If $x=y$, then $2y^2=10$, or $y=\pm\sqrt{5}$ and $x=\pm\sqrt{5}$.

3. From (1) $y=x$ or $-3x$. If $y=x$, $2x+x^2=32$, $x=-1\pm\sqrt{33}$. If $y=-3x$, $x-3x+9x^2=32$, $x=2$, or $-1\frac{7}{9}$, $y=-6$, or $5\frac{1}{3}$.

4. From (2) $y=-x$ or $3x$. If $y=-x$, $2x^2+2x=12$, $x=2$ or -3 and $y=-2$ or 3. If $y=3x$, $10x^2+2x=12$, $x=1$ or $-\frac{6}{5}$, $y=3$, or $-\frac{18}{5}$.

5. From (1) $x=-\frac{1}{2}y$ or $-\frac{3}{2}y$. If $x=-\frac{1}{2}y$, $y^2=1$, $y=\pm 1$, $x=\mp\frac{1}{2}$. If $x=-\frac{3}{2}y$, $9y^2=1$, $y=\pm\frac{1}{3}$ and $x=\mp\frac{3}{2}$.

6. From (1) $x=2y$ or $-7y$. If $x=2y$, $y^2=2y-1$, $y=1$, $x=2$. If $x=-7y$, $y^2+7y+1=0$, from which the other roots follow.

7. From (1) $x=\frac{4}{3}y$, or $\frac{3}{2}y$. If $x=\frac{4}{3}y$, $\frac{16}{9}y^2-\frac{4}{3}y^2-y=1$, $y=3$, or $-\frac{3}{4}$, $x=4$ or -1. If $x=\frac{3}{2}y$, $y=2$ or $-\frac{2}{3}$, $x=3$ or -1.

8. From (2) $y=2x$ or $\frac{3}{4}x$. If $y=2x$, $x^2+4x=5$ and $x=1$ or -5, $y=2$ or -10. If $y=\frac{3}{4}x$, $x^2+\frac{3}{2}x=5$ or $2x^2+3x-10=0$.

9. Four sets of equations may be formed: (1) $x-y=0$, $x+y-6=0$ from which $x=3$, $y=3$; (2) $x-y=0$, $y+3=0$ from which $x=-3$, $y=-3$; (3) $x-2=0$, $x+y-6=0$ from which $x=2$, $y=4$; (4) $x-2=0$, $y+3=0$ from which $x=2$, $y=-3$.

Exercise 133 — Page 284

1. Multiply (1) by 2 and (2) by 7, and subtract to eliminate the absolute terms, then $2x^2 - 7xy + 6y^2 = 0$, or $(x - 2y)(2x - 3y) = 0$. Then $x = 2y$, or $\frac{3}{2}y$. Substitute $x = 2y$ in (1) and $y = \pm 2$, then $x = \pm 4$ Substitute $x = \frac{3}{2}y$ in (1) and $y = \pm 4$, then $x = \pm 6$.

2. When the absolute terms are eliminated we get $(3x - 5y)(x - 6y) = 0$, or $x = \frac{5}{3}y$, or $6y$. Combine $x = \frac{5}{3}y$ with (1) and $y = \pm 3$, $x = \pm 5$. Combine $x = 6y$ with (1) and $y = \pm \frac{1}{3}\sqrt{3}$, $x = \pm 2\sqrt{3}$.

3. Eliminate the absolute terms and $(5x - 4y)(x - 3y) = 0$. Combine $x = 3y$ with (2), and $y = \pm\sqrt{3}$, $x = \pm 3\sqrt{3}$. Combine $x = \frac{4}{5}y$ with (2) and $y = \pm 5$, $x = \pm 4$.

4. Eliminate the absolute terms and $(4x - 7y)(x + 2y) = 0$. Combine $x = \frac{7}{4}y$ with (1) and $y = \pm 4$, $x = \pm 7$. Combine $x = -2y$ with (1) and $y = \pm 1$, $x = \mp 2$.

5. Eliminate the absolute terms and $(5x - 6y)(x + y) = 0$. Combine $x = \frac{6}{5}y$ with (1), and $y = \pm 5$, $x = \pm 6$. On trial $x = -y$ is seen to be impossible.

6. Eliminate the absolute terms and $(x - 3y)(x - y) = 0$. Combine $x = 3y$ with (1) and $y = \pm 3$, $x = \pm 9$. $x = y$ is impossible.

7. Eliminate the absolute terms and $x = \pm 2y$. Substitute $x = 2y$ in (1) and $y = \pm 2$, $x = \pm 4$. $x = -2y$ is impossible.

8. Eliminate the absolute terms and $(x - 3y)(x - y) = 0$. Combine $x = 3y$ with (1) and $y = \pm 1$, $x = \pm 3$. $x = y$ is impossible.

9. Eliminate the absolute terms and $(x - 3y)(3x + 4y) = 0$. Combine $x = 3y$ with (2) and $y = \pm 2$, $x = \pm 6$. Combine $x = -\frac{4}{3}y$ with (2) and $y = \pm 3\sqrt{-1}$, $x = \mp 4\sqrt{-1}$.

10. Eliminate the absolute terms, and $(x - 2y)(6x + 11y) = 0$. Now combine $x = 2y$ and $x = -\frac{11}{6}y$ with equation (1).

11. Eliminate the absolute terms, and $(2x - y)(10x + y) = 0$. Combine $y = 2x$ and $y = -10x$ with equation (2).

12. Eliminate the absolute terms, and $(x + 6y)(11x - 46y) = 0$. Combine $x = -6y$ and $x = \frac{46}{11}y$ with equation (2).

13. Eliminate the absolute terms, and $(x - 4y)(10x - 13y) = 0$. Combine $x = 4y$ and $x = \frac{13}{10}y$ with equation (1).

14. Eliminate the absolute terms, and $(2x - y)(5x + 4y) = 0$. Combine $y = 2x$ and $y = -\frac{5}{4}x$ with equation (1).

15. Multiply (1) by 2 and subtract, and $(x - y)(4x - y) = 0$. Combine $y = x$ and $y = 4x$ with equation (1).

16. Eliminate the absolute terms, and $(x-2y)(5x-y)=0$. Substitute $x=2y$ in (1) and $y^2=10$, or $y=\pm 3.16$. Substitute $y=5x$ in (1) and the resulting equation has no real roots.

17. Let the tens' digit be x and the units' y, then the number is $10x+y$. Then $x(10x+y)=105$, $y(x+y)=40$. Eliminate the absolute terms and $(5x-3y)(16x+7y)=0$. Substitute $x=\frac{3}{5}y$ in (2) and $y=5$, $x=3$.

Exercise 134 — Page 286

1. $x^2+2xy+y^2=64$, then $x^2-2xy+y^2=64-60=4$ or $x-y=\pm 2$ and $x+y=8$; $x=5$ or 3, $y=3$ or 5.

2. $x^2-2xy+y^2=16$, then $x^2+2xy+y^2=16+48=64$ or $x+y=\pm 8$ and $x-y=4$; $x=6$ or -2, $y=2$ or -6.

3. $x^2-2xy+y^2=1$ or $2xy=24$, then $x^2+2xy+y^2=49$ or $x+y=\pm 7$ and $x-y=1$; $x=4$ or -3, $y=3$ or -4.

4. $x^2+2xy+y^2=121$ or $2xy=60$, then $x^2-2xy+y^2=1$ or $x-y=\pm 1$ and $x+y=11$; $x=6$ or 5, $y=5$ or 6.

5. $x^2-2xy+y^2=1$, then $x^2+2xy+y^2=121$ or $x+y=\pm 11$ and $x-y=\pm 1$; $x=\pm 6$ or ± 5, $y=\pm 5$ or ± 6.

6. $x^2-2xy+y^2=64$, then $xy=-7$ and $x^2+2xy+y^2=36$ or $x+y=\pm 6$ and $x-y=8$; $x=7$ or 1, $y=-1$ or -7.

7. $x^2+2xy+y^2=25$, then $xy=6$ and $x^2-2xy+y^2=1$ or $x-y=\pm 1$ and $x+y=5$; $x=3$ or 2, $y=2$ or 3.

8. $xy=30$, $x-y=\pm 7$; $x=10$ or 3, $y=3$ or 10.

9. $x^2+2xy+y^2=1$ or $5x^2+10xy+5y^2=5$, then $xy=-2$, and $x-y=\pm 3$; $x=2$ or -1, $y=-1$ or 2.

10. $x^2+2xy+y^2=169$ or $x+y=\pm 13$ and $x^2-2xy+y^2=9$ or $x-y=\pm 3$; $x=\pm 8$ or ± 5, $y=\pm 5$ or ± 8.

11. $x^2-2xy+y^2=-101+150=49$ or $x-y=\pm 7$ and $x^2+2xy+y^2=169$ or $x+y=\pm 13$; $x=\pm 10$ or ± 3, $y=\pm 3$ or ± 10.

12. $2x^2+4xy+2y^2=2$ or $x+y=\pm 1$ and $2x^2-4xy+2y^2=50$ or $x-y=\pm 5$; $x=\pm 3$ or ± 2, $y=\mp 2$ or ∓ 3.

13. Dividing (1) by (2) $x^2+xy+y^2=19$ and $x^2-2xy+y^2=1$, then $xy=6$ and $x+y=\pm 5$; $x=3$ or -2, $y=2$ or -3.

14. $x^2-xy+y^2=76$, $x-y=\pm 6$; $x=10$ or 4, $y=4$ or 10.

15. $x+y=9$, $xy=14$, $x-y=\pm 5$; $x=7$ or 2, $y=2$ or 7.

16. Divide (1) by (2), then $x^2-xy+y^2=3$ and $xy=2$. Then $x+y=\pm 3$ and $x-y=\pm 1$. (See Art. 198, Ex. 4.)

17. $x^2+xy+y^2=19$, $xy=6$, $x+y=\pm 5$, $x-y=\pm 1$.

18. $x^4 - 2x^2y^2 + y^4 = 9$ or $x^2 - y^2 = \pm 3$, $x^2 + y^2 = \pm 5$.

19. From (1) $x + y = 7$ or -4. Combine $x + y = 7$ with $x - y = 3$ and $x = 5$, $y = 2$. Combine $x + y = -4$ with $x - y = 3$ and $x = -\frac{1}{2}$, $y = -3\frac{1}{2}$.

20. $x - y = 4$ or 3. Then $x + y = \pm 8$ or $\pm\sqrt{57}$.

21. From (1) $xy = 12$ or 15. Then $x - y = \pm 4$ or ± 2.

22. If they are x and y, then $x + y = 17$, $x^2 + y^2 = 169$; $x = 5$, $y = 12$.

23. $x^2 + y^2 = 625$, $xy = 300$, $x + y = 35$, $x - y = 5$; $x = 20$, $y = 15$.

24. $x + y = 12$, $x^2 + y^2 = 72\frac{1}{2}$, $xy = 35\frac{3}{4}$, $x - y = 1$; $x = 6\frac{1}{2}$, $y = 5\frac{1}{2}$.

25. $xy = 270$, $(x - 3)(y - 3) = 180$ or $xy - 3(x + y) + 9 = 180$ or $x + y = 33$. Then $x - y = 3$ and $x = 18$, $y = 15$.

26. $x + y = 10$, $\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$ or $\frac{10}{xy} = \frac{8}{15}$ or $xy = 18\frac{3}{4}$; $x = 7\frac{1}{2}$, $y = 2\frac{1}{2}$.

27. $xy - y + 2x = 11$ or $7\frac{1}{2} - y + 2x = 11$ or $2x - y = 3\frac{1}{2}$. Now substitute $y = 2x - 3\frac{1}{2}$ in $2xy = 15$ and $x = 3$ or $-\frac{5}{4}$, $y = 2\frac{1}{2}$ or -6.

28. If the side of A is x and of B y, then $x^2 - y^2 = 63$, $x - y = 3$. By division $x + y = 21$, then $x = 12$, $y = 9$.

29. $xy = 1$ and $\frac{1}{x} + \frac{1}{y} = \frac{25}{12}$ or $\frac{x + y}{xy} = \frac{25}{12}$ or $x + y = \frac{25}{12}$. Then $x - y = \frac{7}{12}$ and $x = \frac{4}{3}$, $y = \frac{3}{4}$.

30. $x^2 + 2xy + 4y^2 = 28$. Substitute $x = 2 + 2y$ and $y = 1$ or -2.

31. If the tens' digit is x and the units' y, then $x + y = \frac{1}{7}(10x + y)$ or $x = 2y$ and $x^2 + y^2 = 10x + y - 4$. Solving, $y = 4$, $x = 8$.

32. $xy = 1161$ and $x + y = 70$, then $x - y = 16$; $x = 43$, $y = 27$.

33. Divide (2) by (1) and $\frac{1}{x} + \frac{1}{y} = .1$, then $\frac{1}{x} = .2$, $\frac{1}{y} = .1$.

34. If x is the tens and y the units, then $10x + y + 10y + x = 121$ or $x + y = 11$ and $xy = 28$; $x = 7$ or 4, $y = 4$ or 7, and the number is 74 or 47.

35. Let the hyp. $= x$ and the other sides y and z. Then $x + y + z = 24$, $x^2 = y^2 + z^2$, and $yz = 48$. From (2) and (3) $(y + z)^2 = x^2 + 96$ and from (1) $(y + z)^2 = (24 - x)^2$ or $x^2 + 96 = (24 - x)^2$; $x = 10$, $y = 8$, $z = 6$.

36. Let the sides of the first be a and b and of the second c and d, then $a + b = c + d$, $ab = cd$. Solving for a and b in the usual way we find $a - b = \pm(c - d)$ and since $a + b = c + d$, $a = c$ or d, $b = d$ or c. In either case the sides are equal in pairs and therefore the two rectangles are equal in all respects.

37. $xy = 1\frac{1}{4}$, $x + y = \pm 3$, $x - y = \pm 2$; $x = \pm\frac{5}{2}$ or $\pm\frac{1}{2}$, $y = \pm\frac{1}{2}$ or $\pm\frac{5}{2}$.

38. The greatest distance is the length of the diagonal. If the sides are x and y, then $xy = 1200$, $x^2 + y^2 = 2500$, from which $x + y = 70$, $x - y = 10$ and $x = 40$, $y = 30$.

39. If the radii are x and y, then $x + y = 8$ and $\pi(x^2 + y^2) = \frac{2}{3} \times 81\,\pi$ or $x^2 + y^2 = 54$, from which $xy = 5$ and $x - y = \sqrt{44} = 6.633$; $x = 7.32$, $y = .68$.

40. If the length is x and the width y, then $x + y = \frac{1}{2}\,b$, $xy = a$. Substitute $y = \frac{1}{2}\,b - x$ in $xy = a$ and solve for x.

Exercise 135 — Page 290

1. The origin is the centre of each of these circles. In the first the radius is 2 and in the second the radius is 3. In the third the point (2, 3) lies on the circle and in the fourth the point (3, 5) lies on the circle.

2. To draw the graph of $x^2 + y^2 = 13$, describe a circle with its centre at the origin and passing through the point (2, 3). The graph of $x - y = 1$ passes through the points (1, 0) and (0, − 1). The straight line will be seen to cut the circle at the points (3, 2) and (− 2, − 3) and therefore the solutions of the given equations are $x = 3$ or -2, $y = 2$ or -3.

3. The graph of $x^2 + y^2 = 25$ is a circle with centre at the origin and radius 5. The graph of $2\,x + 3\,y = 18$ passes through the points (9, 0) and (0, 6). The two graphs meet at the point (3, 4) which gives $x = 3$, $y = 4$ as a solution. The coördinates of the other point at which they intersect are approximately $x = 2\frac{2}{3}$, $y = 4\frac{1}{3}$.

The graphs of $x^2 + y^2 = 10$ and $2\,x - y = 5$ will be found to intersect at the points (3, 1) and (1, − 3).

4. If the numbers are x and y, then $x + y = 8$ and $x^2 + y^2 = 25$. Draw the graphs of these equations and it will be found that the line does not cut the circle and there are therefore no real roots of these equations. If the line whose equation is $x + y = 7$ is drawn, it will be found to cut the circle at the points (3, 4) and (4, 3).

Exercise 136 — Page 290

1. $x + y = 28$, $x - y = 12$; $x = 20$, $y = 8$.

2. $5\,x - 2\,y = 12$, $5\,x + 2\,y = 8$; $x = 2$, $y = -1$.

3. $x + y = 10$, $x^2 + y^2 = 58$, then $x - y = \pm 4$; $x = 7$ or 3, $y = 3$ or 7.

4. Substitute $x = 2 + \frac{3}{2}\,y$ in equation (2); $y = 2$ or $-\frac{50}{13}$, $x = 5$ or $-\frac{49}{13}$.

5. Substitute $x = \frac{1}{3}(4 + 4\,y)$ in (2) and $y = 2$ or $-\frac{59}{17}$, $x = 4$ or $-\frac{56}{17}$.

6. $x + y = 5$, $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{5}{6}$. Then $xy = 6$; $x = 3$ or 2, $y = 2$ or 3.

7. Substitute $x = 3 - y$ in (1) and $y^2 - 4\,y + 4 = 0$; $y = 2$, $x = 1$.

8. From (1), $x=2y$ or $-3y$. Substitute in (2); $y=\pm 2$ or $\pm 6\sqrt{-1}$, $x=\pm 4$ or $\mp 2\sqrt{-1}$.

9. Let the dimensions of the first be $3x$, $2x$ and of the second $2y$, y, then $6x^2-2y^2=664$, $10x-6y=60$. Solving, $x=12$, $y=10$ and the dimensions are 36, 24 and 20, 10.

10. Eliminate the absolute terms and $(3x-5y)(x+2y)=0$. Combine $x=\frac{5}{3}y$ with (2) and $y=\pm 3$, $x=\pm 5$; $x+2y=0$ is impossible.

11. Eliminate the absolute terms and $(x+2y)(10x-y)=0$. Combine $x=-2y$ with (2) and $y=\pm 2$, $x=\mp 4$. Combine $y=10x$ with (1) and $x=\pm\frac{1}{2}$, $y=\pm 5$.

12. Let x be the length and y the width, then $xy=300$, $(x-2)(y-3)=216$, or $xy-2y-3x+6=216$, or $2y+3x=90$. Combining with $xy=30$, we get $x=20$ or 10, $y=15$ or 30. The length is therefore 20 and the width 15.

13. By division $x=2y$. Combine with (2) and $y=\pm 5$, $x=\pm 10$.

14. Eliminate the absolute terms and $(2x-5y)(7x+3y)=0$. Combine $2x-5y=0$ with (2) and $y=\pm 2$, $x=\pm 5$. Combine $7x+3y=0$ with (2) and $y=\pm\dfrac{7}{\sqrt{2}}$, $x=\mp\dfrac{3}{\sqrt{2}}$.

15. If the length is x and the width y, then $9xy=10{,}800$ and $9(x+10)(y+6)=16{,}200$ or $xy=1200$ and $(x+10)(y+6)=1800$. Solving, $x=50$ or 40, $y=24$ or 30. The dimensions are either 50×24 or 40×30.

16. $x-y=6$, $xy=-5$, $x+y=\pm 4$; $x=5$ or 1, $y=-1$ or -5.

17. Substitute $x=7-y$ in (1) and $y=3$ or $-\frac{17}{7}$, $x=4$ or $\frac{66}{7}$.

18. From (1) $x+y=4$ or 3 and from (2) $xy=4$ or 2. Now solve $x+y=4$, $xy=4$; $x+y=4$, $xy=2$; $x+y=3$, $xy=4$; $x+y=3$, $xy=2$.

19. $xy=28$, $x-y=5$, then $x^2+y^2=(x-y)^2+2xy=81$.

20. By division $4x^2-2xy+y^2=28$. Substitute $y=10-2x$ and $x^2-5x+6=0$; $x=3$ or 2, $y=4$ or 6.

21. $(x+\sqrt{2})^2+x^2=1$ or $2x^2+2x\sqrt{2}+1=0$, $x=-\dfrac{1}{\sqrt{2}}$, $y=\dfrac{1}{\sqrt{2}}$.

22. Let x be the greater and y the less, then $x(x+y)=192$, $y(x-y)=32$; $x=12$, $y=4$. (See Ex. 14, page 290.)

23. $\dfrac{a^2}{x^2}+\dfrac{2ab}{xy}+\dfrac{b^2}{y^2}=16$ or $\dfrac{a}{x}+\dfrac{b}{y}=\pm 4$. Similarly, $\dfrac{a}{x}-\dfrac{b}{y}=\pm 2$.

24. $(3x-5y)(4x-7y)=0$, or $x=\frac{5}{3}y$ or $\frac{7}{4}y$.

25. $xy=6$, $x^2-y^2=5$. Then $5xy=6x^2-6y^2$ or $(3x+2y)(2x-3y)=0$. Combine $2x-3y=0$ and $xy=6$; $x=\pm 3$, $y=\pm 2$.

26. From (1), $\frac{x}{y}=2$ or -3. Combine with (2); $y=4$ or -1, $x=8$ or 3.

27. $x^2+3xy+2y^2=300$, $x^2+2y^2=3xy$, then $xy=50$. Substitute $x=\frac{50}{y}$ in $x^2+2y^2=150$ and $y^2=25$ or 50.

28. $x-y=15$, $x^2+y^2=1625$; $x=35$, $y=20$.

29. Let $\frac{1}{x}=a$, $\frac{2}{y}=b$, then $a+b=8$, $a^2+b^2=40$; $a=2$ or 6, $b=6$ or 2.

30. Let $\frac{1}{x}=a$, $\frac{1}{y}=b$, then $a-b=\frac{1}{12}$, $4a^2+6b^2=\frac{5}{12}$, whence $4(b+\frac{1}{12})^2+6b^2=\frac{5}{12}$; $b=\frac{1}{6}$ or $-\frac{7}{30}$, $a=\frac{1}{4}$ or $-\frac{3}{20}$.

31. $x-y=15$, $\frac{1}{2}xy=y^3$ or $x=2y^2$; $y=3$ or $-\frac{5}{2}$, $x=18$ or $12\frac{1}{2}$.

32. $10x^2-37xy+30y^2=0$ or $(2x-5y)(5x-6y)=0$. Combine $x=\frac{5}{2}y$ and $x=\frac{6}{5}y$ with $xy=10$; $x=\pm 5$ or $\pm\frac{6}{\sqrt{3}}$, $y=\pm 2$ or $\pm\frac{5}{\sqrt{3}}$.

33. $xy+4=4y$, $xy-3=3x$, then $4y-3x=7$. Substitute $y=\frac{1}{4}(7+3x)$ in (2); $x=3$ or $-\frac{4}{3}$, $y=4$ or $\frac{3}{4}$.

34. Suppose the faster walks a mile in x min., then the other takes $(x+18)$ min. Together they walk $\frac{1}{x}+\frac{1}{x+18}$ of a mile in 1 min., then $300\left(\frac{1}{x}+\frac{1}{x+18}\right)=25$ from which $x=18$, or the faster rate is $3\frac{1}{3}$ mi. per hour and the slower $1\frac{2}{3}$.

35. Let $\frac{1}{x}=a$, $\frac{1}{y}=b$, then $a+b=\frac{1}{3}$, $a^2+9b^2=\frac{1}{8}$. Solving, $a=\frac{1}{4}$ or $\frac{7}{20}$, $b=\frac{1}{12}$ or $-\frac{1}{60}$, whence $x=4$ or $2\frac{6}{7}$, $y=12$ or -60.

36. From (1), $x+y=5$ or -4. Now solve $x+y=5$, $xy=6$ and $x+y=-4$, $xy=6$.

37. If they are x and $x+2$, then $(x+2)^3-x^3=218$; $x=5$ or -7.

38. By division $x^2-3xy+4y^2=2$, then $xy=2$ and $x^2+4xy+4y^2=16$ or $x+2y=\pm 4$. Similarly, $x-2y=0$, then $x=\pm 2$, $y=\pm 1$.

39. $x^2+y-y^2-x=0$ or $(x-y)(x+y-1)=0$. Now combine $x=y$ with $x^2+y=3$ and $x=1-y$ with $x^2+y=3$.

40. If they are x and y, then $x-y=s$ and $x^2+y^2=d^2$. Substitute $x=y+s$ in $x^2+y^2=d^2$ and solve by the formula. When $s=7$ and $d=13$, $x=12$ and $y=5$.

41. $(3x+y)^2-21(3x+y)+104=0$ or $3x+y=8$ or 13. Now solve $3x+y=8$, $xy=4$ and $3x+y=13$, $xy=4$.

42. $(x+2y)^2-18(x+2y)+80=0$ or $x+2y=8$ or 10.

43. Multiply (2) by 3 and add, then $(x+y)^3=216$ or $x+y=6$. Then from (2) $xy=5$.

Exercise 138 — Page 296

1. a^{10}. **2.** x^2. **3.** x^5. **4.** a^4b^6. **5.** 81. **6.** 1. **7.** ab^2. **8.** 25. **9.** 64. **10.** 36. **11.** -1. **12.** $\dfrac{1}{ab}$. **13.** x^{a+b+c}. **14.** a^{2x}. **15.** x^{3m}. **16.** x^{2b}. **17.** $a^4b^6c^8$. **18.** $x^{2a+2b+2c}$.

20. The sum of the indices is $4a + 4b$.

22. $\dfrac{x^{2a}}{x^{b+c}} \times \dfrac{x^{2b}}{x^{c+a}} \times \dfrac{x^{2c}}{x^{a+b}} = \dfrac{x^{2a+2b+2c}}{x^{2a+2b+2c}} = 1.$

25. $4^n = (2^2)^n = 2^{2n}$; $9^6 = (3^2)^6 = 3^{12}$.

26. $27^5 = (3^3)^5 = 3^{15}$; $9^7 = (3^2)^7 = 3^{14}$; $3^{15} \div 3^{14} = 3$.

27. $\dfrac{2^n \times 2^{n-1} \times 2^2}{2^{2n}} = \dfrac{2^{2n+1}}{2^{2n}} = 2$; $\dfrac{3^{2n} \times 3^{n+5}}{3^{3n+3}} = \dfrac{3^{3n+5}}{3^{3n+3}} = 9.$

28. $5^{2x+1} = 5^{x+3}$, then $2x + 1 = x + 3$ or $x = 2$; $2^{2x} = 2^{x+7}$, then $2x = x + 7$; $3^{6x+6} = 3^{3x+15}$, then $6x + 6 = 3x + 15$; $2^x \cdot 2^{2x} \cdot 2^{3x} = 2^{12x-12}$, then $x + 2x + 3x = 12x - 12$.

Exercise 139 — Page 300

1. $\sqrt{a}$. **2.** $\sqrt[3]{x}$. **3.** $\sqrt[5]{y}$. **4.** $\sqrt[3]{a^2}$. **5.** $\sqrt[5]{x^4}$. **6.** 1. **7.** $\dfrac{1}{a^2}$. **8.** $\dfrac{1}{x}$. **9.** $\dfrac{1}{x^4}$. **10.** $\dfrac{1}{y^n}$. **11.** $\dfrac{1}{\sqrt{m}}$. **12.** $\dfrac{1}{\sqrt[3]{x^2}}$. **13.** 3. **14.** 2. **15.** 5. **16.** 10. **17.** 8. **18.** 9. **19.** $\frac{1}{8}$. **20.** $\frac{1}{25}$. **21.** $\frac{1}{1000}$. **22.** $\frac{1}{2}$. **23.** $\frac{1}{2}$. **24.** 1. **25.** $\frac{1}{4}$. **26.** 1. **27.** $\frac{1}{2}$. **28.** .064. **29.** $1\frac{7}{9}$. **30.** $\frac{1}{16}$. **31.** 1. **32.** 4. **33.** $\dfrac{a^2}{b^3}$. **34.** $\dfrac{2}{a^3}$. **35.** $\dfrac{b^3}{a^2}$. **36.** $\dfrac{ad^4}{cb^3}$. **37.** $\dfrac{x^3y^4}{a^2b^3}$. **38.** $\dfrac{x^5}{y}$. **39.** $\dfrac{2y^2}{3x}$. **40.** $\dfrac{9x^3y^3}{64}$. **41.** $2xyz^{-2}$. **42.** $4a^3b^{-3}$. **43.** $3a^2xb^{-4}$.

44. $5a^{-2}(c+d)^2$. **45.** $\dfrac{1}{16^{\frac{3}{4}}} = \dfrac{1}{(\sqrt[4]{16})^3} = \dfrac{1}{8}$. **46.** $32^{\frac{3}{5}} = (\sqrt[5]{32})^3 = 2^3 = 8$.

47. $(\frac{1}{25})^{-2} = 25^2 = 625$. **48.** $\dfrac{1}{(\sqrt[3]{.027})^2} = \dfrac{1}{(.3)^2} = 11\frac{1}{9}$.

49. $25^{\frac{3}{2}} = (\sqrt{25})^3 = 125$. **50.** $\dfrac{1}{(\sqrt[3]{-8})^2} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$.

51. $\dfrac{1}{\sqrt[3]{8^4}} = \dfrac{1}{2^4} = \dfrac{1}{16}$. **52.** $16^{\frac{5}{4}} = (\sqrt[4]{16})^5 = 32$. **53.** $\dfrac{\frac{1}{2}}{\frac{1}{4} - \frac{1}{8}} = \dfrac{\frac{1}{2}}{\frac{1}{8}} = 4$.

54. $\sqrt{\frac{50}{98}} = \sqrt{\frac{25}{49}} = \frac{5}{7}$. **55.** $\dfrac{1}{\sqrt{\frac{9}{16}}} = \dfrac{1}{\frac{3}{4}} = \dfrac{4}{3}$. **56.** $\dfrac{\frac{1}{27} - \frac{1}{8}}{\frac{1}{3} - \frac{1}{2}} = \dfrac{-\frac{19}{216}}{-\frac{1}{6}} = \dfrac{19}{36}$.

57. $(\sqrt[4]{\frac{16}{81}})^3 = (\frac{2}{3})^3 = \frac{8}{27}$. **58.** $\dfrac{16^{-\frac{3}{4}}a^3}{81^{-\frac{3}{4}}b^{-3}} = \dfrac{81^{\frac{3}{4}}a^3b^3}{16^{\frac{3}{4}}} = \dfrac{27a^3b^3}{8}$.

59. $x = 4^2 = 16$; $x^{\frac{1}{3}} = \sqrt[5]{32} = 2$, $x = 8$; $x^{\frac{1}{4}} = 3$, $x = 81$; $x^{-1} = 9$, $x = \frac{1}{9}$; $x^{-\frac{1}{2}} = 2$, $x^{-1} = 4$, $x = \frac{1}{4}$.

Exercise 140 — Page 302

1.
$$\begin{array}{l}
x^{\frac{1}{2}} + 3 \\
\underline{x^{\frac{1}{2}} - 2} \\
x + 3x^{\frac{1}{2}} \\
\underline{\quad - 2x^{\frac{1}{2}} - 6} \\
x + x^{\frac{1}{2}} - 6
\end{array}$$

2.
$$\begin{array}{l}
x + x^{\frac{1}{2}} + 1 \\
\underline{x^{\frac{1}{2}} - 1} \\
x^{\frac{3}{2}} + x + x^{\frac{1}{2}} \\
\underline{\quad - x - x^{\frac{1}{2}} - 1} \\
x^{\frac{3}{2}} \qquad\qquad - 1
\end{array}$$

3.
$$\begin{array}{l}
x^{\frac{3}{2}} - x + x^{\frac{1}{2}} - 1 \\
\underline{x^{\frac{1}{2}} + 1} \\
x^2 - x^{\frac{3}{2}} + x - x^{\frac{1}{2}} \\
\underline{\quad + x^{\frac{3}{2}} - x + x^{\frac{1}{2}} - 1} \\
x^2 \qquad\qquad\qquad - 1
\end{array}$$

5.
$$\begin{array}{l}
a^{\frac{1}{2}} - 1 + 2a^{-\frac{1}{2}} \\
\underline{a^{\frac{1}{2}} + 1 - 2a^{-\frac{1}{2}}} \\
a - a^{\frac{1}{2}} + 2 \\
\quad + a^{\frac{1}{2}} - 1 + 2a^{-\frac{1}{2}} \\
\underline{\qquad - 2 + 2a^{-\frac{1}{2}} - 4a^{-1}} \\
a \qquad - 1 + 4a^{-\frac{1}{2}} - 4a^{-1}
\end{array}$$

6.
$$\begin{array}{l}
a - a^{\frac{1}{2}} + 1 \\
\underline{a - a^{\frac{1}{2}} + 1} \\
a^2 - a^{\frac{3}{2}} + a \\
\quad - a^{\frac{3}{2}} + a - a^{\frac{1}{2}} \\
\underline{\qquad\qquad a - a^{\frac{1}{2}} + 1} \\
a^2 - 2a^{\frac{3}{2}} + 3a - 2a^{\frac{1}{2}} + 1
\end{array}$$

7.
$$\begin{array}{l}
x + 5x^{\frac{3}{4}} + 6x^{\frac{1}{2}} \\
\underline{x^{\frac{1}{4}} - 1 - x^{-\frac{1}{4}}} \\
x^{\frac{5}{4}} + 5x + 6x^{\frac{3}{4}} \\
\quad - x - 5x^{\frac{3}{4}} - 6x^{\frac{1}{2}} \\
\underline{\qquad\quad - x^{\frac{3}{4}} - 5x^{\frac{1}{2}} - 6x^{\frac{1}{4}}} \\
x^{\frac{5}{4}} + 4x \qquad - 11x^{\frac{1}{2}} - 6x^{\frac{1}{4}}
\end{array}$$

9.
$$\begin{array}{l}
x + x^{\frac{1}{2}}y^{\frac{1}{2}} + y \\
\underline{x - x^{\frac{1}{2}}y^{\frac{1}{2}} + y} \\
x^2 + x^{\frac{3}{2}}y^{\frac{1}{2}} + xy \\
\quad - x^{\frac{3}{2}}y^{\frac{1}{2}} - xy - x^{\frac{1}{2}}y^{\frac{3}{2}} \\
\underline{\qquad\qquad + xy + x^{\frac{1}{2}}y^{\frac{3}{2}} + y^2} \\
x^2 \qquad + xy \qquad + y^2
\end{array}$$

11.
$$\begin{array}{l|l|l}
\underline{a^{\frac{1}{2}} + 2b^{\frac{1}{2}}} & a + 5a^{\frac{1}{2}}b^{\frac{1}{2}} + 6b & \underline{a^{\frac{1}{2}} + 3b^{\frac{1}{2}}} \\
 & \underline{a + 2a^{\frac{1}{2}}b^{\frac{1}{2}}} & \\
 & \quad 3a^{\frac{1}{2}}b^{\frac{1}{2}} + 6b & \\
 & \quad \underline{3a^{\frac{1}{2}}b^{\frac{1}{2}} + 6b} &
\end{array}$$

12.
$$\begin{array}{l|l}
\underline{x^2+2+2x^{-2}} & x^3-x^2+x-2-2x^{-2}-2x^{-3} \;\big|\; \underline{x-1-x^{-1}} \\
 & \underline{x^3 \quad +2x \quad +2x^{-1}} \\
 & -x^2-x-2-2x^{-1}-2x^{-2}-2x^{-3} \\
 & \underline{-x^2 \quad -2 \quad -2x^{-2}} \\
 & -x \quad -2x^{-1} \quad -2x^{-3} \\
 & \underline{-x \quad -2x^{-1} \quad -2x^{-3}}
\end{array}$$

13.
$$\begin{array}{l|l}
\underline{x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}} & x^{\frac{4}{3}}+x^{\frac{2}{3}}y^{\frac{2}{3}}+y^{\frac{4}{3}} \;\big|\; \underline{x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}} \\
 & \underline{x^{\frac{4}{3}}+xy^{\frac{1}{3}}+x^{\frac{2}{3}}y^{\frac{2}{3}}} \\
 & -xy^{\frac{1}{3}} \\
 & \underline{-xy^{\frac{1}{3}}-x^{\frac{2}{3}}y^{\frac{2}{3}}-x^{\frac{1}{3}}y} \\
 & x^{\frac{2}{3}}y^{\frac{2}{3}}+x^{\frac{1}{3}}y+y^{\frac{4}{3}} \\
 & \underline{x^{\frac{2}{3}}y^{\frac{2}{3}}+x^{\frac{1}{3}}y+y^{\frac{4}{3}}}
\end{array}$$

14.
$$\begin{array}{l|l}
\underline{1-x^{\frac{1}{3}}+3x^{\frac{2}{3}}} & 1-5x^{\frac{1}{3}}-x \;\big|\; \underline{1-4x^{\frac{1}{3}}-7x^{\frac{2}{3}}+4x} \\
 & \underline{1-x^{\frac{1}{3}}+3x^{\frac{2}{3}}} \\
 & -4x^{\frac{1}{3}}-3x^{\frac{2}{3}}-x \\
 & \underline{-4x^{\frac{1}{3}}+4x^{\frac{2}{3}}-12x} \\
 & -7x^{\frac{2}{3}}+11x \\
 & \underline{-7x^{\frac{2}{3}}+7x-21x^{\frac{4}{3}}} \\
 & +4x+21x^{\frac{4}{3}}
\end{array}$$

15. The square roots evidently are $a^{\frac{1}{2}}+3$ and $5x-x^{-1}$.

16.
$$\begin{array}{r|l}
 & a^2+4a^{\frac{3}{2}}+6a+4a^{\frac{1}{2}}+1 \;\big|\; \underline{a+2a^{\frac{1}{2}}+1} \\
 & \underline{a^2} \\
\underline{2a+2a^{\frac{1}{2}}} & 4a^{\frac{3}{2}}+6a+4a^{\frac{1}{2}}+1 \\
 & \underline{4a^{\frac{3}{2}}+4a} \\
\underline{2a+4a^{\frac{1}{2}}+1} & 2a+4a^{\frac{1}{2}}+1 \\
 & \underline{2a+4a^{\frac{1}{2}}+1}
\end{array}$$

17.
$$\begin{array}{r|l}
 & 4x^{\frac{5}{3}}-20x^{\frac{4}{3}}+37x-30x^{\frac{2}{3}}+9x^{\frac{1}{3}} \;\big|\; \underline{2x^{\frac{5}{6}}-5x^{\frac{1}{2}}+3x^{\frac{1}{6}}} \\
 & \underline{4x^{\frac{5}{3}}} \\
\underline{4x^{\frac{5}{6}}-5x^{\frac{1}{2}}} & -20x^{\frac{4}{3}}+37x \\
 & \underline{-20x^{\frac{4}{3}}+25x} \\
\underline{4x^{\frac{5}{6}}-10x^{\frac{1}{2}}+3x^{\frac{1}{6}}} & 12x-30x^{\frac{2}{3}}+9x^{\frac{1}{3}} \\
 & \underline{12x-30x^{\frac{2}{3}}+9x^{\frac{1}{3}}}
\end{array}$$

18.
$$\begin{array}{rl}
 & 25x^{\frac{4}{3}} - 30x^{\frac{2}{3}} + 49 - 24x^{-\frac{2}{3}} + 16x^{-\frac{4}{3}} \,\big|\, \underline{5x^{\frac{2}{3}} - 3 + 4x^{-\frac{2}{3}}} \\
 & \underline{25x^{\frac{4}{3}}} \\
\underline{10x^{\frac{2}{3}} - 3} \,\big| & -30x^{\frac{2}{3}} + 49 \\
 & \underline{-30x^{\frac{2}{3}} + 9} \\
\underline{10x^{\frac{2}{3}} - 6 + 4x^{-\frac{2}{3}}} \,\big| & 40 - 24x^{-\frac{2}{3}} + 16x^{-\frac{4}{3}} \\
 & \underline{40 - 24x^{-\frac{2}{3}} + 16x^{-\frac{4}{3}}}
\end{array}$$

19. It is true since $(a^{\frac{1}{3}} + b^{\frac{1}{3}})(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}) = a + b$.

21.
$$\begin{array}{rl}
\underline{2a^m - 7} \,\big| & 10a^{3m} - 27a^{2m} - 32a^m + 14 \,\big|\, \underline{5a^{2m} + 4a^m - 2} \\
 & \underline{10a^{3m} - 35a^{2m}} \\
 & 8a^{2m} - 32a^m \\
 & \underline{8a^{2m} - 28a^m} \\
 & -4a^m + 14 \\
 & \underline{-4a^m + 14}
\end{array}$$

22. $(x + x^{\frac{1}{2}} + 1)^2 = x^2 + 2x^{\frac{3}{2}} + 3x + 2x^{\frac{1}{2}} + 1$.
$(x - x^{\frac{1}{2}} + 1)^2 = x^2 - 2x^{\frac{3}{2}} + 3x - 2x^{\frac{1}{2}} + 1$.

23. $(\sqrt{a} + 1)(\sqrt{a} - 1) = a - 1$, $(\sqrt{3a} + \sqrt{2})(\sqrt{3a} - \sqrt{2}) = 3a - 2$.

24.
$$\begin{array}{rl}
 & x^2 - 4x^{\frac{3}{2}} + 10x - 12x^{\frac{1}{2}} + 9 \,\big|\, \underline{x - 2x^{\frac{1}{2}} + 3} \\
 & \underline{x^2} \\
\underline{2x - 2x^{\frac{1}{2}}} \,\big| & -4x^{\frac{3}{2}} + 10x \\
 & \underline{-4x^{\frac{3}{2}} + 4x} \\
\underline{2x - 4x^{\frac{1}{2}} + 3} \,\big| & 6x - 12x^{\frac{1}{2}} + 9 \\
 & \underline{6x - 12x^{\frac{1}{2}} + 9}
\end{array}$$

Exercise 141 — Page 304

2. $(a^{\frac{1}{2}} + a^{-\frac{1}{2}})^2 - 1 = a + 2 + a^{-1} - 1 = a + 1 + a^{-1}$.

3. $(x - x^{\frac{1}{2}} - 1)^2 = x^2 + x + 1 - 2x^{\frac{3}{2}} - 2x + 2x^{\frac{1}{2}}$.
$(2a - 2 - a^{-1})^2 = 4a^2 + 4 + a^{-2} - 8a - 4 + 4a^{-1}$.

4. The cube of $a^{\frac{1}{2}} + 1$ is the same as $(x + 1)^3$ where $x = a^{\frac{1}{2}}$.

5. The product of the first two $= (x + y)^2 - xy = x^2 + xy + y^2$.

6. This is similar to dividing $a^3 + b^3$ by $a^2 - ab + b^2$.

7. The first $= (a^{\frac{1}{2}} + b^{\frac{1}{2}})^2 - c = (a^{\frac{1}{2}} + b^{\frac{1}{2}} + c^{\frac{1}{2}})(a^{\frac{1}{2}} + b^{\frac{1}{2}} - c^{\frac{1}{2}})$.

10. $\dfrac{(x^{\frac{1}{2}}-2)(x^{\frac{1}{2}}+3)}{(x^{\frac{1}{2}}-2)(x^{\frac{1}{2}}-3)}$, $\dfrac{(a^{\frac{1}{3}}-b^{\frac{1}{3}})(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}})}{a^{\frac{1}{3}}-b^{\frac{1}{3}}}$,

$\dfrac{(a+\sqrt{ab}+b)(a-\sqrt{ab}+b)}{a+\sqrt{ab}+b}$.

11. The first term in the cube root is $x^{\frac{1}{2}}$ and the last is -2 so the cube root is $x^{\frac{1}{2}}-2$. The first term is x and the trial divisor for finding the next term is $3x^2$ so that the second term $=-x^{\frac{1}{2}}$. The last term is $+1$.

Exercise 142 — Page 305

5. $\sqrt[3]{64}=4$, $\sqrt[4]{81^3}=3^3=27$. **8.** $4+1-2-3+0+4=4$.

9. $3^{-\frac{1}{4}}\times 3^{\frac{3}{4}}=3^{\frac{1}{2}}=1.732$. **10.** $8+2-\frac{1}{2}-\frac{1}{8}$, $2-4+\frac{1}{8}+\frac{1}{16}$.

11. $5^{\frac{3}{8}+\frac{3}{4}+\frac{7}{8}}=5^2=25$; $16^{\frac{5}{17}+\frac{1}{34}+\frac{3}{17}}=16^{\frac{1}{2}}=4$.

12. $x^{\frac{1}{2}}=2$, $x=4$; $2^x\cdot 2^{2x}=64$ or $2^{3x}=2^6$, $3x=6$ or $x=2$.

13. $2^{\frac{4}{3}}\times 2^{-\frac{4}{3}}\times 3^{-\frac{4}{3}}\times 3^{\frac{1}{3}}=2^0\times 3^{-1}=\frac{1}{3}$; $(4+8)\times\frac{1}{8}=1\frac{1}{2}$.

14. $\dfrac{2^{n+1}\times 2^{-n}\times 3^{-n}}{5^{n+1}\times 3^{-n}\times 5^{-n}}=\dfrac{2}{5}$; $\dfrac{3.2^n-2.2^n}{10.2^{n-1}-3.2^{n-1}}=\dfrac{2^n}{7.2^{n-1}}=\dfrac{2}{7}$.

15. $\dfrac{(\sqrt{a}+3)(\sqrt{a}+5)}{(\sqrt{a}+3)(\sqrt{a}+4)}$, $\dfrac{(x^{\frac{1}{4}}+1)(3x^{\frac{1}{4}}+2)}{x^{\frac{1}{4}}+1}$, $\dfrac{a(a^{\frac{1}{2}}+b)}{b(a^{\frac{1}{2}}+b)(a^{\frac{1}{2}}-b)}$.

16.
$$\begin{array}{l}
x^{\frac{3}{2}}y^{\frac{1}{2}}-2xy+4x^{\frac{1}{2}}y^{\frac{3}{2}}\\
\underline{\quad x^{\frac{1}{2}}+2y^{\frac{1}{2}}\qquad\qquad\quad}\\
x^2y^{\frac{1}{2}}-2x^{\frac{3}{2}}y+4xy^{\frac{3}{2}}\\
\underline{\qquad +2x^{\frac{3}{2}}y-4xy^{\frac{3}{2}}+8x^{\frac{1}{2}}y^2}\\
x^2y^{\frac{1}{2}}\qquad\qquad\qquad\quad +8x^{\frac{1}{2}}y^2
\end{array}$$

17. The product $=(x^{\frac{1}{5}}y^{-\frac{1}{2}}+x^{-\frac{1}{5}}y^{\frac{1}{2}})^2-1=x^{\frac{2}{5}}y^{-1}+2+x^{-\frac{2}{5}}y-1$.

18.
$$\begin{array}{l}
\underline{x^{\frac{4}{3}}+x^{\frac{2}{3}}y^{-\frac{2}{3}}+y^{-\frac{4}{3}}}|\,x^2 \qquad\qquad\qquad -y^{-2}\,|\underline{x^{\frac{2}{3}}-y^{-\frac{2}{3}}}\\
\qquad\qquad\qquad\qquad \underline{x^2+x^{\frac{4}{3}}y^{-\frac{2}{3}}+x^{\frac{2}{3}}y^{-\frac{4}{3}}\qquad\quad}\\
\qquad\qquad\qquad\qquad\quad -x^{\frac{4}{3}}y^{-\frac{2}{3}}-x^{\frac{2}{3}}y^{-\frac{4}{3}}-y^{-2}\\
\qquad\qquad\qquad\qquad\quad -x^{\frac{4}{3}}y^{-\frac{2}{3}}-x^{\frac{2}{3}}y^{-\frac{4}{3}}-y^{-2}
\end{array}$$

19. This is similar to dividing a^4-b^4 by $a-b$, or by factoring:

$$a^{4m}-b^{4m}=(a^{2m}+b^{2m})(a^m+b^m)(a^m-b^m).$$

20. Since dividend = quotient × divisor + remainder, subtract the remainder from the dividend and divide by the quotient.

21. The first term in the square root of $x^2 - 4x + 2 + 4x^{-1} + x^{-2}$ is x. The trial divisor is $2x$, so that the second term is -2 and the last must be $-x^{-1}$.

23. $xy + x - y = (a^2 + 1)(a^{-2} + 1) + a^2 + 1 - (a^{-2} + 1) = 2 + 2a^2$. Similarly, $xy - x + y = 2a^{-2} + 2 = a^{-2}(2 + 2a^2)$.

24. $.2^4 = .0016$; $(1.2)^2 = 1.44$; $(1.5)^3 = 3.375$; $.5^{-3} = 2^3 = 8$.

25. $2x = a^{\frac{1}{2}} + a^{-\frac{1}{2}}$, $2y = a^{\frac{1}{2}} - a^{-\frac{1}{2}}$, $4xy = a - a^{-1}$.

$x^2 + 2xy + y^2 = a$, $x^2 - 2xy + y^2 = a^{-1}$, $2x^2 + 2y^2 = a + a^{-1}$.

26. $4a^2 = 2^{2x} + 2 + 2^{-2x}$, $4b^2 = 2^{2x} - 2 + 2^{-2x}$, $4a^2 - 4b^2 = 4$.

27. $e^{2x} - 2 + e^{-2x} + 4 = (e^x + e^{-x})^2$.

The first term is $x^{\frac{3}{2}}$ and the trial divisor is $2x^{\frac{3}{2}}$, so that the second term is $-2xy^{\frac{1}{2}}$. The last term is $\pm y^{\frac{3}{2}}$ and the trial divisor is $\pm 2y^{\frac{3}{2}}$ so that the next to last term is $\mp 3x^{\frac{1}{2}}y$. It is now easily determined by squaring whether the result is $x^{\frac{3}{2}} - 2xy^{\frac{1}{2}} + 3x^{\frac{1}{2}}y - y^{\frac{3}{2}}$ or $x^{\frac{3}{2}} - 2xy^{\frac{1}{2}} - 3x^{\frac{1}{2}}y + y^{\frac{3}{2}}$.

28. $a^{\frac{3}{4}}b^{\frac{1}{2}}c^{-\frac{1}{3}} \times a^{-\frac{1}{4}}c^{-\frac{1}{3}}b^{\frac{1}{3}} \times ca^{\frac{1}{2}}b^{\frac{1}{6}}c^{\frac{2}{3}} = a^1b^1c^1 = abc$.

30. $10^{.60206} = (10^{.30103})^2 = 2^2 = 4$; $10^{1.50515} = 2^5 = 32$.

31. $7^{2.0104} \times 7^{2.0593} = 7^{4.0697} = 50 \times 55 = 2750$.

32. $\dfrac{2^{n+1}}{2^{n^2-n}} \times \dfrac{2^{n^2-1}}{2^{2n+2}} = \dfrac{2^{n^2+n}}{2^{n^2+n+2}} = \dfrac{1}{2^2} = \dfrac{1}{4}$.

33. $3.3^x + 2^y = 35$, $3^x + 4.2^y = 41$. Solving, $3^x = 9$, $2^y = 8$.

34. $x^{\frac{1}{3}} - x^{-\frac{1}{3}} \,\big|\, x + 2x^{\frac{5}{6}} - 2x^{\frac{1}{6}} + 2x^{-\frac{1}{6}} - 2x^{-\frac{5}{6}} - x^{-1}$

$$
\begin{array}{l}
x - x^{\frac{1}{3}} \\
\hline
2x^{\frac{5}{6}} + x^{\frac{1}{3}} - 2x^{\frac{1}{6}} + 2x^{-\frac{1}{6}} - 2x^{-\frac{5}{6}} - x^{-1} \\
2x^{\frac{5}{6}} - 2x^{\frac{1}{6}} \\
\hline
x^{\frac{1}{3}} + 2x^{-\frac{1}{6}} - 2x^{-\frac{5}{6}} - x^{-1} \\
x^{\frac{1}{3}} - x^{-\frac{1}{3}} \\
\hline
+2x^{-\frac{1}{6}} + x^{-\frac{1}{3}} - 2x^{-\frac{5}{6}} - x^{-1} \\
2x^{-\frac{1}{6}} - 2x^{-\frac{5}{6}} \\
\hline
x^{-\frac{1}{3}} - x^{-1} \\
x^{-\frac{1}{3}} - x^{-1} \\
\hline
\end{array}
$$

The quotient $= x^{\frac{2}{3}} + 2x^{\frac{1}{2}} + 1 + 2x^{-\frac{1}{2}} + x^{-\frac{2}{3}}$.

35. $x\sqrt{x} = \frac{9}{4} \times \frac{3}{2} = \frac{27}{8}$; $(\frac{9}{4})^{\frac{27}{8}} = (\frac{3}{2})^{\frac{27}{4}}$ and $(\frac{27}{8})^{\frac{9}{4}} = (\frac{3}{2})^{\frac{27}{4}}$.

36. When the expression is written with fractional indices and in descending powers of x it is $4x^2 + 4x^{\frac{3}{2}} - 4x^{\frac{3}{4}} + x - 2x^{\frac{1}{4}} + x^{-\frac{1}{2}}$. The first term in the square root is $2x$, the trial divisor is $4x$ so that the second term is $x^{\frac{1}{2}}$ and then the last term must be $-x^{-\frac{1}{4}}$.

Exercise 143 — Page 309

1. $3\sqrt{3}$, $10\sqrt{a}$, $b\sqrt{5}$, $2a\sqrt{2b}$, $4x\sqrt{2xy}$, $11a^2b\sqrt{3}$.

2. $2\sqrt[3]{2}$, $2a\sqrt[3]{a}$, $3x\sqrt[3]{2x}$, $5a\sqrt[3]{b}$, $-3a\sqrt[3]{3}$, $-\frac{1}{2}a\sqrt[3]{a}$.

3. $2\sqrt[4]{2}$, $3\sqrt[4]{3}$, $\frac{1}{2}a\sqrt[4]{a}$, $2\sqrt[5]{2}$, $2(x+y)\sqrt{2}$.

4. $\sqrt{12}$, $\sqrt{200}$, $\sqrt{9a}$, $\sqrt{5a^2}$, $\sqrt{a^2b^3}$, $\sqrt{(a-b)^3}$.

5. $\sqrt[3]{24}$, $\sqrt[3]{189}$, $\sqrt[3]{-2}$, $\sqrt[3]{a^5b}$, $\sqrt[3]{xy}$, $\sqrt[4]{80}$.

6. $\sqrt{a^2-b^2}$, $\sqrt{\dfrac{m+n}{m-n}}$, $\sqrt{\dfrac{x^2-xy}{xy+y^2}}$.

7. $2^{\frac{1}{3}} = 2^{\frac{2}{6}} = \sqrt[6]{4}$, $3^{\frac{1}{2}} = 3^{\frac{3}{6}} = \sqrt[6]{27}$; $2^{\frac{1}{3}} = 2^{\frac{4}{12}} = \sqrt[12]{16}$, $3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{27}$.

8. $\sqrt{18}$ or $\sqrt{12}$; $\sqrt{150}$ or $\sqrt{147}$; $\sqrt[6]{125}$ or $\sqrt[6]{100}$; $\sqrt[3]{1.26^3}$ or $\sqrt[3]{2}$; $\sqrt[12]{81}$ or $\sqrt[12]{125}$. **9.** $2\sqrt{2} + 3\sqrt{2} + 7\sqrt{2}$. **10.** $10\sqrt{5} + 4\sqrt{5} - 2\sqrt{5}$.

11. $12\sqrt{2} + 25\sqrt{2} - 4\sqrt{2}$. **12.** $2\sqrt[3]{2} - 4\sqrt[3]{2} + 5\sqrt[3]{2}$.

13. $2\sqrt[3]{12} + 2\sqrt[3]{12} + 3\sqrt[3]{12}$. **14.** $2\sqrt[4]{2} + 3\sqrt[4]{2} + 5\sqrt[4]{2}$.

15. $5\sqrt{3} - 6\sqrt{3} + 50\sqrt{3} + 8\sqrt{3} - 49\sqrt{3} + \sqrt{3}$.

16. $x\sqrt{x+y} + x\sqrt{x+y} - (x+y)\sqrt{x+y} - (x-y)\sqrt{x+y}$.

18. $2\sqrt[3]{2}$, $3\sqrt[3]{2}$, $10\sqrt[3]{2}$, $\frac{1}{2}\sqrt[3]{2}$, $\frac{1}{10}\sqrt[3]{2}$, $\sqrt[3]{2}$.

19. $2^1 \times 2^{\frac{1}{2}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} = 2^2 = 4$.

Exercise 144 — Page 310

1. $6\sqrt{15}$. **2.** $ax\sqrt{6}$. **3.** $x\sqrt{y}$. **4.** $\sqrt[3]{20}$. **5.** $14\sqrt{6}$. **6.** 2. **7.** $\sqrt[3]{a^4-b^4}$. **8.** $(x-4)(x-9) = x^2 - 13x + 36$. **9.** $(\sqrt{2}+\sqrt{3})^2 - (\sqrt{5})^2 = 2 + 3 + 2\sqrt{6} - 5 = 2\sqrt{6}$. **10.** $(a^{\frac{1}{3}} - 1)(a^{\frac{1}{3}} - 2)(a^{\frac{1}{3}} + 3) = a - 7a^{\frac{1}{3}} + 6$. **11.** $\sqrt{36-11} = 5$. **12.** $(3\sqrt{2} + 2\sqrt{3} + 2\sqrt{2})^2 = (5\sqrt{2} + 2\sqrt{3})^2 = 62 + 20\sqrt{6}$. **13.** $\sqrt{2}$, 6. **14.** $\sqrt{5}$, 10. **15.** $\sqrt{2}$, 8. **16.** 1, 8. **17.** $\sqrt{2}$, 32. **18.** $\sqrt[3]{4}$, 2. **19.** $\sqrt[3]{9}$, -3. **20.** $\sqrt[4]{27}$, 6. **21.** $3 + \sqrt{2}$, 7. **22.** $\sqrt{a} - b$, $a - b^2$. **23.** $3\sqrt{2} + 2\sqrt{3}$, 6. **24.** $a\sqrt{b} + b\sqrt{a}$, $a^2b - b^2a$. **25.** $\dfrac{15\sqrt{10}}{5}$.

26. $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$. **27.** $\sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$. **28.** $\frac{(4+2\sqrt{2})(2\sqrt{2}-2)}{(2\sqrt{2}+2)(2\sqrt{2}-2)}$
$= \frac{4\sqrt{2}}{4}$. **29.** $\frac{\sqrt{8}-\sqrt{3}}{(\sqrt{8}+\sqrt{3})(\sqrt{8}-\sqrt{3})} = \frac{2\sqrt{2}-\sqrt{3}}{5}$.

30. $\frac{a^2(\sqrt{a^2+b^2}+b)}{a^2+b^2-b^2} = \sqrt{a^2+b^2}+b$. **31.** $\frac{(a+b-c)(\sqrt{a+b}-\sqrt{c})}{a+b-c}$.

32. $\frac{(\sqrt{x+y}-\sqrt{x-y})^2}{(x+y)-(x-y)} = \frac{2x-2\sqrt{x^2-y^2}}{2y}$.

33. $\frac{22}{5+\sqrt{14}} = \frac{22(5-\sqrt{14})}{25-14} = 10-2\sqrt{14} = 10-7.483 = 2.517$.

34. $x = 7-4\sqrt{3}$, $y = 7+4\sqrt{3}$, $x^2+y^2 = 49-56\sqrt{3}+48+49$
$+56\sqrt{3}+48 = 194$.

35. $(2\sqrt{3}-\sqrt{2})^3 = 36\sqrt{3}-38\sqrt{2}$; $(\sqrt{3}-\sqrt{2})^3 = 9\sqrt{3}-11\sqrt{2}$.

36. $\frac{3-\sqrt{5}}{3+\sqrt{5}} = \frac{7-3\sqrt{5}}{2}$, $\frac{4+\sqrt{5}}{4-\sqrt{5}} = \frac{21+8\sqrt{5}}{11}$, $\frac{5+\sqrt{5}}{1+\sqrt{5}} = \sqrt{5}$, $\frac{5+2\sqrt{5}}{2+\sqrt{5}} = \sqrt{5}$.

37. $\frac{7-2\sqrt{5}}{4-\sqrt{5}} = \frac{18-\sqrt{5}}{11}$, $\frac{15+6\sqrt{5}}{2+\sqrt{5}} = 3\sqrt{5}$.

38. $(3-\sqrt{7})^3-5(3-\sqrt{7})^2-4(3-\sqrt{7})+2 = 90-34\sqrt{7}-80$
$+30\sqrt{7}-12+4\sqrt{7}+2 = 0$.

39. If each dimension is x, the diagonal on the floor is $x\sqrt{2}$ and from the floor to the farthest corner at the ceiling is $x\sqrt{3}$. If $x\sqrt{3} = 18$, $x = \frac{18}{\sqrt{3}} = 6\sqrt{3} = 10.392$ ft. $= 10$ ft. 4.7 in.

Exercise 145 — Page 314

1. $2x-5=9$, $x=7$. **2.** $3x-2=4(x-2)$, $x=6$.

3. $2x^{\frac{1}{2}}=4$, $x=4$. **4.** $5x-7=8$, $x=3$. **5.** $8x=27$, $x=3\frac{3}{8}$.

6. $3x-25=8$, $x=11$. **7.** $8(x-7)=x-14$, $x=6$.

8. $\sqrt{x+5}=5-\sqrt{x}$. Squaring, $x+5=25-10\sqrt{x}+x$ or $\sqrt{x}=2$, $x=4$.

9. $\sqrt{x+45}=9-\sqrt{x}$, $x+45=81-18\sqrt{x}+x$ or $\sqrt{x}=2$, $x=4$.

10. $1+2\sqrt{x+2}+x+2=x$ or $2\sqrt{x+2}=-3$, $4x+8=9$, $x=\frac{1}{4}$. If we substitute $\frac{1}{4}$ for x in the first side of the equation, we get $1+\sqrt{\frac{9}{4}}$ $=1+\frac{3}{2}=\frac{5}{2}$. But when $x=\frac{1}{4}$, the second side is $\frac{1}{2}$, which shows that $\frac{1}{4}$ is an extraneous root.

11. $\sqrt{x+15}=11-\sqrt{x+4}$, $x+15=121-22\sqrt{x+4}+x+4$, $x=21$.

12. $4x^2+3x-16=4x^2+8x+4$, $x=-4$. This root is extraneous.

13. $(x+5)(x-3)=(x-1)^2$, $x=4$.

14. $7\sqrt{x}+10=4(2\sqrt{x}-2)$, $x=324$.

15. $(\sqrt{x}-3)(\sqrt{x}-2)=(\sqrt{x}+1)(\sqrt{x}+3)$, $x-5\sqrt{x}+6=x+4\sqrt{x}+3$, $x=\frac{1}{9}$.

16. Solving in the usual way, we get $x=5$, which does not satisfy the equation. It is evidently a root of $\sqrt{x+4}+\sqrt{x-4}=4$.

18. $\sqrt{x+8}=\sqrt{x+3}+2\sqrt{x}$, $x+8=x+3+4\sqrt{x^2+3x}+4x$ or $5-4x=4\sqrt{x^2+3x}$, $25-40x+16x^2=16x^2+48x$, $88x=25$.

19–20. See Ex. 15. **21.** $x^2-a^2+b^2=x^2+a^2+b^2-2ax+2bx-2ab$ or $2ax-2bx=2a^2-2ab$, $x=a$.

22. Multiply each side by $\sqrt{x-9}$, $\sqrt{x^2-9x}+x-9=36$ or $\sqrt{x^2-9x}=45-x$, $x^2-9x=2025-90x+x^2$, $x=25$.

24. Solving, we get $x=\frac{25}{88}$ which is an extraneous root. See Ex. 18 and Ex. 10.

25. $x+4a-2\sqrt{x^2+4ax}+x=4(b+x)$ or $2a-2b-x=\sqrt{x^2+4ax}$.

26. $x^3-6x^2+11x-5=x^3-6x^2+12x-8$, $x=3$.

27. Cubing, $125(70x+29)=729(14x-15)$, $x=10$.

28. $\sqrt{x+6}+\sqrt{x-6}=3\sqrt{x+6}-3\sqrt{x-6}$, $2\sqrt{x-6}=\sqrt{x+6}$, $x=10$.

29. Change $\dfrac{5x-1}{\sqrt{5x}+1}$ into $\sqrt{5x}-1$, then $\sqrt{5x}=3$, $x=\frac{9}{5}$.

30. Apply the theorem in Art. 185 and $\dfrac{\sqrt{a+x}}{\sqrt{a-x}}=\dfrac{c+1}{c-1}$ or $\dfrac{a+x}{a-x}=\dfrac{c^2+2c+1}{c^2-2c+1}$. Again, by Art. 185, $\dfrac{a}{x}=\dfrac{c^2+1}{2c}$.

Exercise 146 — Page 316

1. $\sqrt{x}=20-x$, $x=400-40x+x^2$, $x^2-41x+400=0$, $(x-16)(x-25)=0$, $x=16$ or 25. On verifying, 16 is a root, but 25 is extraneous.

2. The solution is the same as in Ex. 1. The correct root is 25, the other being extraneous.

3. $3x-5=9-6\sqrt{x-2}+x-2$, $3\sqrt{x-2}=6-x$, $x=3$ or 18. $x=3$ verifies, but 18 is extraneous.

4. Same as Ex. 3, 18 is the root, and 3 is extraneous.

5. $5x^2+11=9x^2+30x+25$, $x=-7$ or $-\frac{1}{2}$, and $-\frac{1}{2}$ is extraneous.

6. Same as Ex. 5, $x=-\frac{1}{2}$ is the root, and -7 is extraneous.

7. $2=5\sqrt{x}-2x$, $25x=4+8x+4x^2$, $x=4$ or $\frac{1}{4}$.

8. $4(7x+4)=9x^2-90x+225$, $x=11$ or $\frac{19}{9}$; $\frac{19}{9}$ is extraneous.

9. $\dfrac{x+16+4-x}{\sqrt{64-12x-x^2}}=\dfrac{5}{2}$, $x^2+12x=0$; $x=0$ or -12.

10. $x^3 + 2x^2 - 10x + 5 = x^3 - 3x^2 + 3x - 1$; $x = 2$ or $\frac{3}{5}$.

11. $x + a = a - b + x + b - 2\sqrt{(a-b)(x+b)}$, $x = -b$.

12. $2x + 5 = 4 + x - 1 + 4\sqrt{x-1}$, $x^2 - 12x + 20 = 0$; $x = 2$ or 10.

13. $16(x^2 + x + 3) = 9(2x^2 + 5x - 2)$; $x = 2$ or $-\frac{33}{2}$.

14. $7\sqrt{2 - x^2} = x$, $x = \pm\frac{7}{5}$, of which $-\frac{7}{5}$ is extraneous.

15. $6\sqrt{3x^2 + 4} = 8\sqrt{2x^2 + 1}$, $x^2 = 4$; $x = \pm 2$.

16. $\sqrt{x+7} + \sqrt{x-7} = 7\sqrt{2}$, $x + 7 + x - 7 + 2\sqrt{x^2 - 49} = 98$; $x = 25$.

17. Multiply (1) by 2 and (2) by 3 and subtract, $\sqrt{x} = 3$, $\sqrt{y} = 2$.

18. From (1), $\sqrt{xy} = 6$ or -5 of which -5 is extraneous. Now solve $xy = 36$ and $x + y = 13$, $x = 9$ or 4, $y = 4$ or 9.

19. Adding, $x + y = 20$, $\sqrt{xy} = 8$ or $xy = 64$.

20. From (1), $\sqrt{x+y} = 5$ or -6. From (2), $\sqrt{x-y} = 3$ or -4. Now solve $x + y = 25$, $x - y = 9$, the negative value being extraneous.

21. Divide (1) by (2) and $x - \sqrt{xy} + y = 7$, then $x + y = 10$, $\sqrt{xy} = 3$ or $xy = 9$.

22. $x + \sqrt{xy} + y = 3\frac{1}{2}$, then $x + y = 2\frac{1}{2}$, $\sqrt{xy} = 1$.

23. Let $\sqrt{x^2 - 3x + 6} = y$, then $y^2 - y - 2 = 0$ or $y = 2$ or -1, of which -1 is impossible. Now $x^2 - 3x + 6 = 4$.

24. Let $\sqrt{x^2 - x - 6} = y$, then $y^2 - y = 30$ or $y = 6$ or -5, then $x^2 - x - 6 = 36$.

25. $\dfrac{x+y}{\sqrt{xy}} = \dfrac{5}{2}$. Since $x + y = 10$, $\sqrt{xy} = 4$ or $xy = 16$.

Exercise 147 — Page 318

1. $\sqrt{2} + \sqrt{3}$. **2.** $\sqrt{6} - \sqrt{2}$. **3.** $\sqrt{3} + 1$. **4.** $2 - \sqrt{2}$. **5.** $2 + \sqrt{6}$. **6.** $\sqrt{7} + 2\sqrt{2}$. **7.** $\sqrt{7} + 1$. **8.** $2 - \sqrt{3}$. **9.** $2 + \sqrt{5}$. **10.** $\sqrt{x} + \sqrt{y}$. **11.** $\sqrt{x} - \sqrt{y}$. **12.** $\sqrt{x+y} + \sqrt{x-y}$. **13.** $2\sqrt{2} - \sqrt{7}$. **14.** $\sqrt{10} - 2\sqrt{2}$. **15.** $\sqrt{15} + \sqrt{5}$. **16.** $3\sqrt{2}$. **17.** $3\sqrt{5} + \sqrt{2}$. **18.** $3\sqrt{6} - \sqrt{3}$. **19.** $1 - \sqrt{a-1}$. **20.** $\sqrt{2x+1} + \sqrt{2x-1}$. **21.** $1 - \sqrt{a-b-1}$.

Exercise 148 — Page 319

1. $94 - 2\sqrt{2205}$. Let the two factors of 2205 be a and b, then $a + b = 94$, $ab = 2205$. Then $a^2 - 2ab + b^2 = 8836 - 8820 = 16$ or $a - b = 4$. Then $a = 49$, $b = 45$. The square root $= \sqrt{49} - \sqrt{45} = 7 - 3\sqrt{5}$.

2. $38 + 2\sqrt{360}$. $a + b = 38$, $ab = 360$; $a = 20$, $b = 18$ and the square root $= \sqrt{20} + \sqrt{18} = 2\sqrt{5} + 3\sqrt{2}$.

3. $47 - 2\sqrt{540}$. $a + b = 47$, $ab = 540$; $a = 27$, $b = 20$ and the square root $= \sqrt{27} - \sqrt{20} = 3\sqrt{3} - 2\sqrt{5}$.

4. $107 - 2\sqrt{2160}$. $a + b = 107$, $ab = 2160$, $(a - b)^2 = 11449 - 8640 = 2809$ or $a - b = 53$; $a = 80$, $b = 27$. The root $= \sqrt{80} - \sqrt{27} = 4\sqrt{5} - 3\sqrt{3}$.

5. $94 + 2\sqrt{2205}$. See Ex. 1. The root $= 7 + 3\sqrt{5}$.

6. $101 - 2\sqrt{2548}$. $a = 52$, $b = 49$. The root $= \sqrt{52} - \sqrt{49} = 2\sqrt{13} - 7$.

7. $67 + 2\sqrt{882}$. $a = 49$, $b = 18$. The root $= 7 + 3\sqrt{2}$.

8. $28 - 2\sqrt{75}$. $a = 25$, $b = 3$. The root $= \sqrt{25} - \sqrt{3} = 5 - \sqrt{3}$.

9. $xy + 2\sqrt{xy^3 - y^4}$. $a + b = xy$, $ab = xy^3 - y^4$. Then $a^2 - 2ab + b^2 = x^2y^2 - 4xy^3 + 4y^4$ or $a - b = xy - 2y^2$. Then $a = xy - y^2$, $b = y^2$, and the root $= \sqrt{xy - y^2} + y$.

10. The root of $16 - 6\sqrt{7}$ is $3 - \sqrt{7}$; $\dfrac{1}{3 - \sqrt{7}} = \dfrac{3 + \sqrt{7}}{2} = 2.8229$.

11. The root $= \dfrac{\sqrt{3} - 1}{2 + \sqrt{3}} = \dfrac{(\sqrt{3} - 1)(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = 3\sqrt{3} - 5 = .196$.

12. $\sqrt{2}(7 + 4\sqrt{3}) = \sqrt{2}(2 + \sqrt{3})^2$; $\sqrt{5}(2\sqrt{5} + 6) = \sqrt{5}(\sqrt{5} + 1)^2$;
$\sqrt{3}(7 - 4\sqrt{3}) = \sqrt{3}(2 - \sqrt{3})^2$; $\sqrt{2}(59 + 30\sqrt{2}) = \sqrt{2}(5\sqrt{2} + 3)^2$.

13. (1) The roots are $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$.
(2) $17 + 12\sqrt{2} + 17 - 12\sqrt{2} + 2\sqrt{289 - 288} = 34 + 2 = 36 = 6^2$.

14. The root of $49 + 8\sqrt{3} = 1 + 4\sqrt{3}$. The root of $13 + 4\sqrt{3} = 1 + 2\sqrt{3}$. The root of $4 + 2\sqrt{3} = 1 + \sqrt{3}$.

15. The root of $\dfrac{3 + 2\sqrt{2}}{2} = \dfrac{1 + \sqrt{2}}{\sqrt{2}}$; the root of $\dfrac{11 + 4\sqrt{7}}{3} = \dfrac{2 + \sqrt{7}}{\sqrt{3}}$.

16. By Ex. 1, the root $= 7 - 3\sqrt{5} = 7 - 6.7$ which is less than unity and therefore its square is also less than unity.

17. $x^2 = \dfrac{21 - 8\sqrt{5}}{14 - 6\sqrt{5}} = \dfrac{(4 - \sqrt{5})^2}{(3 - \sqrt{5})^2}$, then $x = \dfrac{4 - \sqrt{5}}{3 - \sqrt{5}} = \dfrac{7 + \sqrt{5}}{4} = 2.309$.

18. If the hyp. $= x$, then $x^2 = (\sqrt{5})^2 + (3 + 2\sqrt{2})^2 = 22 + 12\sqrt{2} = (2 + 3\sqrt{2})^2$.

Exercise 149 — Page 322

1. $2\sqrt{-1}, 4\sqrt{-1}, 9\sqrt{-1}, a\sqrt{-1}, 25\sqrt{-1}, 3x^2\sqrt{-1}, (a - b)\sqrt{-1}$.

2. $-1, -a, 1, -1$. **3.** $\sqrt{-1} + 2\sqrt{-1} + 3\sqrt{-1} = 6\sqrt{-1}$.

4. $(5 + 10 + 7)\sqrt{-1} = 22\sqrt{-1}$.

5. $4 + 3\sqrt{-1} + 2 - 4\sqrt{-1} = 6 - \sqrt{-1}$.

6. $a + b\sqrt{-1} + a - b\sqrt{-1} = 2a$.

7. $\sqrt{-1} \times 2\sqrt{-1} = 2(\sqrt{-1})^2 = -2$. **8.** -50. **9.** $-ab$.

10. $3\sqrt{-3} + 10\sqrt{-3} - 8\sqrt{-3} + 20\sqrt{-3} = 25\sqrt{-3}$.

11. $(3 + 5\sqrt{-1})(3 - 5\sqrt{-1}) = 9 + 25 = 34$;
$(5 - 3\sqrt{-1})(5 + 3\sqrt{-1}) = 34$.

12. $16 - 24\sqrt{-1} - 9 + 4 + 24\sqrt{-1} - 36 = -25$.

13. $\dfrac{2}{1-\sqrt{-1}} = \dfrac{2(1+\sqrt{-1})}{(1-\sqrt{-1})(1+\sqrt{-1})} = \dfrac{2(1+\sqrt{-1})}{1+1} = 1 + \sqrt{-1}$.

14. $\dfrac{(-1+\sqrt{-3})(-1+\sqrt{-3})}{(-1-\sqrt{-3})(-1+\sqrt{-3})} = \dfrac{1-2\sqrt{-3}-3}{1+3} = \dfrac{-1-\sqrt{-3}}{2}$.

15. $a^2 + 2ab\sqrt{-1} - b^2 + a^2 - 2ab\sqrt{-1} - b^2 = 2a^2 - 2b^2$.

16. $\frac{1}{4}(-1+\sqrt{-3})^2 = \frac{1}{4}(1 - 2\sqrt{-3} - 3) = \frac{1}{2}(-1-\sqrt{-3})$.

17. Its square $= \frac{1}{4}(1 - 2\sqrt{-3} - 3) = \frac{1}{2}(-1-\sqrt{-3})$.
Its cube $= \frac{1}{2}(-1-\sqrt{-3})\frac{1}{2}(-1+\sqrt{-3}) = \frac{1}{4}(1+3) = 1$.

18. $(2+\sqrt{-3})^2 - 4(2+\sqrt{-3}) + 7 = 4 + 4\sqrt{-3} - 3 - 8 - 4\sqrt{-3} + 7 = 0$.
Therefore $2 + \sqrt{-3}$ is a root. Similarly $2 - \sqrt{-3}$ is a root.

19. $a + b = 4, ab = 13, a^2 + b^2 = -5 + 12\sqrt{-1} - 5 - 12\sqrt{-1} = -10$, and these are all real quantities. (See Art. 228.)

Exercise 150 — Page 324

1. Let the parts be x and $10-x$, then $x(10-x) = 40$ or $x^2 - 10x + 40 = 0$ or $x = 5 \pm \sqrt{-15}$, which is impossible.

2. Let the width be x and length $2x$, then $(2x+10)(x-1) = 4x^2$ or $x^2 - 4x + 5 = 0$ or $x = 2 \pm \sqrt{-1}$ which is impossible. In the second case $(2x+10)(x+1) = 4x^2$ or $x^2 - 6x - 5 = 0$ or $x = 3 \pm \sqrt{14}$. It is now possible, the width being $3 + \sqrt{14}$ and the length $6 + 2\sqrt{14}$.

3. $x(8-x) = 20$ or $x^2 - 8x + 20 = 0$ or $x = 4 \pm 2\sqrt{-1}$, which is impossible.

4. $x^2 - 12x + 36 + a = 0$ or $x = 6 \pm \sqrt{-a}$. Since a is not negative this is impossible unless a is zero.

Exercise 151 — Page 324

3. The roots are $3 + \sqrt{5}$, $\sqrt{21} - 2$, $\sqrt{15} + \sqrt{7}$, $3 - \sqrt{2}$, $3 - \sqrt{3}$.

4. The simplified values are $\dfrac{\sqrt{5}+\sqrt{3}}{2}$, $\sqrt{2}+\sqrt{3}$, $\dfrac{3+7\sqrt{3}-7\sqrt{2}-\sqrt{6}}{2}$, $\dfrac{4\sqrt{10}-2}{3}$.

5. $x = 9$ or 14, the latter being extraneous.

7. $\sqrt{x^2 - 4x} + x - 4 = 8$, $x^2 - 4x = (12-x)^2$; $x = 7\frac{1}{5}$.

8. $6x+7-2\sqrt{6x^2+19x+14}+x+2=2x+2$. $\therefore 5x+7=2\sqrt{6x^2+19x+14}$ or $x^2-6x-7=0$.

9. $x+5+\sqrt{x^2-11x-80}=35$ or $x^2-11x-80=(30-x)^2$.

10. $19+x-\sqrt{285-4x-x^2}=24$ or $285-4x-x^2=(x-5)^2$, from which $x=13$ or -10, of which -10 is extraneous.

11. $(\sqrt{5}+\sqrt{3})^2-2=6+2\sqrt{15}$; $(x+y)^2-4(x+y)$.

12. $\dfrac{6+\sqrt{3}}{2\sqrt{3}}+\dfrac{3\sqrt{3}-2}{4}=\dfrac{3}{\sqrt{3}}+\dfrac{1}{2}+\dfrac{3\sqrt{3}}{4}-\dfrac{1}{2}=\sqrt{3}+\dfrac{3}{4}\sqrt{3}=\dfrac{7}{4}\sqrt{3}$.

13. $2a+\sqrt{b}+2a-\sqrt{b}+2\sqrt{4a^2-b}=4a+2\sqrt{4a^2-b}$.

14 $(p+x-q)^2=2px+x^2$ or $2qx=p^2-2pq+q^2$.

15. $(2\sqrt{3}+3\sqrt{2}+\sqrt{30})(\sqrt{2}+\sqrt{3}-\sqrt{5})=2\sqrt{6}+6+2\sqrt{15}+6+3\sqrt{6}+3\sqrt{10}-2\sqrt{15}-3\sqrt{10}-5\sqrt{6}=12$.

16. Prod. of the first two $=(x-1)^2-2=x^2-2x-1$. Prod. of the last two $=(x+1)^2-3=x^2+2x-2$.

17. When the denominators are rationalized we get $\sqrt{2}(\sqrt{6}-\sqrt{3})-\sqrt{6}(\sqrt{3}-1)+\sqrt{6}(\sqrt{3}-\sqrt{2})=2\sqrt{3}-\sqrt{6}-3\sqrt{2}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}=0$.

18. $\dfrac{1}{x}=\dfrac{1}{a+\sqrt{a^2-1}}=a-\sqrt{a^2-1}$. Then $x+\dfrac{1}{x}=2a$; $x^2+\dfrac{1}{x^2}=a^2+a^2-1+2a\sqrt{a^2-1}+a^2+a^2-1-2a\sqrt{a^2-1}=4a^2-2$; $x^3+\dfrac{1}{x^3}=a^3+3a^2\sqrt{a^2-1}+3a(a^2-1)+(a^2-1)\sqrt{a^2-1}+a^3-3a^2\sqrt{a^2-1}+3a(a^2-1)-(a^2-1)\sqrt{a^2-1}=8a^3-6a$.

19. $3\sqrt{3}-2\sqrt{2}+(3+2\sqrt{2})-(3\sqrt{3}-1)=4$.
$(3+\sqrt{2})+(4-\sqrt{3})+(\sqrt{3}-\sqrt{2})=7$.

20. $\dfrac{3+\sqrt{3}}{\sqrt{3}+1}=\sqrt{3}$; $\dfrac{m+n+m-n}{\sqrt{m^2-n^2}}=\dfrac{2m}{\sqrt{m^2-n^2}}$.

21. $x^2=(-1+2\sqrt{-1})^2=-3-4\sqrt{-1}$;
$x^4=(-3-4\sqrt{-1})^2=-7+24\sqrt{-1}$.

23. Let $x^2+3x-3=y^2$, then $2y^2-y=45$ or $y=5$ or $-4\frac{1}{2}$. If $y=5$, $x^2+3x-3=25$, $x=4$ or -7. The other value of y leads to extraneous roots.

24. $(10+2\sqrt{15}+2\sqrt{10}+2\sqrt{6})+(10+2\sqrt{15}-2\sqrt{10}-2\sqrt{6})+(10-2\sqrt{15}+2\sqrt{10}-2\sqrt{6})+(10-2\sqrt{15}-2\sqrt{10}+2\sqrt{6})=40$.

25. Let $x^2-3x+5=y^2$, then $3y^2-4=4y$ or $y=2$ or $-\frac{2}{3}$. If $y=2$, $x^2-3x+5=4$ or $x=\frac{1}{2}(3\pm\sqrt{5})=2.62$ or $.38$.

26. When the denominators are rationalized we get

$$\frac{a+\sqrt{a^2-b^2}}{b}-\frac{a-\sqrt{a^2-b^2}}{b}=\frac{2\sqrt{a^2-b^2}}{b}.$$

27. $\frac{1}{3+\sqrt{7}}+\frac{1}{3-\sqrt{7}}=\frac{3-\sqrt{7}+3+\sqrt{7}}{2}=3.$

28. If $a+\sqrt{b}=x+y+2\sqrt{xy}$, then $x+y=a$, $4xy=b$. Then $(x-y)^2=a^2-b$. If x and y are to be rational, $x-y$ must be and therefore a^2-b must be a perfect square.

29. $\frac{1}{4}(a^{2x}+2+a^{-2x})-\frac{1}{4}(a^{2x}-2+a^{-2x})=1.$

30. $(\sqrt{2}-1)^3+(\sqrt{2}+1)^3=(5\sqrt{2}-7)+(5\sqrt{2}+7)=10\sqrt{2}.$

31. $x^2=4+2\sqrt{3}$, $x^3=(4+2\sqrt{3})(\sqrt{3}+1)=6\sqrt{3}+10.$

Exercise 152 — Page 329

1. 7, 12. **2.** 5, -11. **3.** -6, 1. **4.** 5, 3. **5.** 4, $-2\frac{1}{3}$.
6. $4\frac{1}{4}$, 1. **7.** $\frac{b}{a}$, $-\frac{c}{a}$. **8.** $\frac{b+c}{a}$, 1. **9.** 0, $-\frac{q}{p}$. **10.** 0, 1.
11. $1\frac{1}{3}$, -2. **12.** $\frac{1}{a+b}$, $a-b$. **13.** 3, 6, 8, 10; 9, 10.

14. Yes, since their sum is 9 and their product is 20.

16. Correct. **17.** Incorrect, since the sum should be -3.

18. Correct. **19.** Incorrect, as the product should be 27.

20. Incorrect both as to sum and product. **21.** Correct.

22. Their sum is 2, therefore the other is -39.

23. By showing that the sum $=3.293$ and the product $=2.439$.

24. The product of the roots must be 1, or $a \div 6=1$ and $a=6$.

25. $m^2-9=0$ or $m=\pm 3$, then $m^3 \div m=9$.

Exercise 153 — Page 330

9. $(x-a)^2=0.$ **10.** $(x+a)(x+b)=0.$ **11.** $x(x-3)=0.$

12. $x(x-m)=0.$ **13.** $(x-3)(x-4)(x-5)=0.$

14. $(x-2)(x-3)(x+1)=0.$ **15.** $(x-a)(x-b)(x-c)=0.$

16. $x(x-a)(x-b)=0.$

17. Sum of the roots $=2m$, prod. $=m^2-n^2$, then the equation is $x^2-2mx+m^2-n^2=0.$

18. Sum $=4a$, prod. $=4a^2-b^2$, equation is $x^2-4ax+4a^2-b^2=0.$

19. Sum $=6$, prod. $=6$, equation is $x^2-6x+6=0.$

20. Sum $=-\frac{1}{2}$, prod. $=-\frac{63}{16}$, equation is $x^2+\frac{1}{2}x-\frac{63}{16}=0.$

21. The equation is $(x+2)(x+4)(x-6)=0$ or $x^3-28x-48=0$.

22. The equation is $(x-\frac{1}{2})(x-\frac{1}{3})(x-\frac{1}{4})=0$ or $(2x-1)(3x-1)(4x-1)=0$.

23. $1.25+4.64=5.89$, $1.25\times 4.64=5.8$, thus they are correct.

24. If $a+b=7$, $a^2-b^2=14$, then $a-b=2$ and $a=4\frac{1}{2}$, $b=2\frac{1}{2}$ and $ab=\frac{45}{4}$. The equation is $x^2-7x+\frac{45}{4}=0$.

25. $a^2+2ab+b^2=49$, then $ab=12$. The equation is $x^2-7x+12=0$.

26. Sum $=\dfrac{2a^2+2b^2}{a^2-b^2}$, prod. $=1$, the equation is $x^2-\dfrac{2a^2+2b^2}{a^2-b^2}x+1=0$.

Sum $=4$, prod. $=\frac{9}{4}$, the equation is $x^2-4x+\frac{9}{4}=0$.

27. $x^2-9x+7=0$, sum $=9$, prod. $=7$; $x^2-x(a+b)=0$, sum $=a+b$, prod. $=0$; $x^2-2px+pq=0$, sum $=2p$, prod. $=pq$; $x^2+x(2a+2b-2c)+a^2+b^2-c^2=0$, sum $=2c-2a-2b$, prod. $=a^2+b^2-c^2$.

28. Since 5 is a root, $x-5$ must be a factor. Divide by x and $x-5$ and factor the quotient.

Then $x(x-5)(x+1)(x+4)=0$; $x=0, 5, -1, -4$.

29. Their sum is 12, so they must be 4 and 8, then $a=32$.

30. Their prod. is 48, so they must be ± 4 and ± 12, then $p=\pm 16$.

Exercise 154 — Page 334

1. $m+n=5$, $mn=3$, then $\dfrac{1}{m}+\dfrac{1}{n}=\dfrac{m+n}{mn}=\dfrac{5}{3}$; $\dfrac{m}{n}+\dfrac{n}{m}=\dfrac{m^2+n^2}{mn}=\dfrac{(m+n)^2-2mn}{mn}=\dfrac{19}{3}$; $m^2+mn+n^2=(m+n)^2-mn=22$.

2. $m+n=7$, $mn=1$, then $m^2+n^2=(m+n)^2-2mn=47$. $m+n=\frac{4}{3}$, $mn=\frac{5}{3}$, then $m^2+n^2=\frac{16}{9}-\frac{10}{3}=-\frac{14}{9}$.

3. $p+q=-\dfrac{2}{3}$, $pq=-2$, then $\dfrac{p}{q}+\dfrac{q}{p}=\dfrac{(p+q)^2-2pq}{pq}=-\dfrac{20}{9}$; $\dfrac{p^2+q^2}{p^2q^2}=\dfrac{(p+q)^2-2pq}{p^2q^2}=\dfrac{10}{9}$; $p^2-pq+q^2=(p+q)^2-3pq=\dfrac{58}{9}$.

4. $m+n=\frac{3}{2}$, $mn=2$, then $m^3+n^3=(m+n)^3-3mn(m+n)=-\frac{45}{8}$. $m+n=1$, $mn=a$, then $m^3+n^3=1-3a$.

5. (1) The roots are 4, 5. The new roots are 8, 10. Then the new equation is $x^2-18x+80=0$. (2) If m, n are the roots, $m+n=9$, $mn=20$, then $2m+2n=18$, $2m\times 2n=80$.

6. (1) The roots are 4, 7. The new roots are 1, 4. Then the new equation is $x^2 - 5x + 4 = 0$. (2) $m + n = 1$, $mn = -1$. Sum of roots of new equation $= m - 3 + n - 3 = -5$, prod. $= (m-3)(n-3) = mn - 3(m+n) + 9 = 5$. The new equation is $x^2 + 5x + 5 = 0$.

7. (1) The roots are $\frac{3}{2}$, -2. The new roots are $\frac{2}{3}$, $-\frac{1}{2}$. Then the new equation is $6x^2 - x - 2 = 0$. (2) $m + n = p$, $mn = q$. Sum of new roots $= \frac{1}{m} + \frac{1}{n} = \frac{m+n}{mn} = \frac{p}{q}$, prod. $= \frac{1}{mn} = \frac{1}{q}$, then the new equation is $x^2 - \frac{px}{q} + \frac{1}{q} = 0$ or $qx^2 - px + 1 = 0$.

8. $m + n = \frac{2}{3}$, $mn = \frac{5}{3}$. (1) $\frac{1}{m} + \frac{1}{n} = \frac{2}{5}$, $\frac{1}{mn} = \frac{3}{5}$. The equation is $x^2 - \frac{2}{5}x + \frac{3}{5} = 0$. (2) $\frac{m}{n} + \frac{n}{m} = \frac{(m+n)^2 - 2mn}{mn} = -\frac{26}{15}$, $\frac{m}{n} \times \frac{n}{m} = 1$. The equation is $x^2 + \frac{26}{15}x + 1 = 0$. (3) $m^2 + n^2 = -\frac{26}{9}$, $m^2n^2 = \frac{25}{9}$. The equation is $x^2 + \frac{26}{9}x + \frac{25}{9} = 0$.

9. $m + n = -a$, $mn = b$, then $m^2 + n^2 = (m+n)^2 - 2mn = a^2 - 2b$; $m^3 + n^3 = (m+n)^3 - 3mn(m+n) = -a^3 + 3ab$.

10. $m + n = -p$, $mn = -q$. Sum of new roots $= m^2 + n^2 = p^2 + 2q$, prod. of roots $= m^2n^2 = q^2$. New equation is $x^2 - x(p^2 + 2q) + q^2 = 0$.

11. $m + n = -\frac{b}{a}$, $mn = \frac{c}{a}$. Sum of new roots $= m + n + 2h = -\frac{b}{a} + 2h$, prod. of roots $= (m+h)(n+h) = mn + h(m+n) + h^2 = \frac{c}{a} - \frac{hb}{a} + h^2$. New equation is $x^2 - x\left(2h - \frac{b}{a}\right) + \frac{c}{a} - \frac{hb}{a} + h^2 = 0$.

12. $m + n = -1$, $mn = -\frac{1}{4}$. Sum of new roots $= \frac{1}{m} + \frac{1}{n} = 4$, product $= \frac{1}{mn} = -4$. New equation is $x^2 - 4x - 4 = 0$.

13. $m + n = p$, $mn = q$, so the equation whose roots are $m + n$ and mn or p and q is $x^2 - x(p + q) + pq = 0$.

14. $m^2 + 2mn + n^2 = 36$, then $mn = 8$ and the required equation is $x^2 + 6x + 8 = 0$.

15. $m + n = -p$, $mn = q$, then $m + 2n + 2m + n = -3p$ and $(m + 2n)(2m + n) = 2(m^2 + n^2) + 5mn = 2(m+n)^2 + mn = 2p^2 + q$, which shows that $m + 2n$, $n + 2m$ are the roots.

16. $p + q = -\frac{b}{a}$, $pq = \frac{c}{a}$, then $p^2 + q^2 = (p+q)^2 - 2pq = \frac{b^2}{a^2} - \frac{2c}{a}$, then $p^4 + p^2q^2 + q^4 = (p^2 + q^2)^2 - p^2q^2 = \left(\frac{b^2}{a^2} - \frac{2c}{a}\right)^2 - \frac{c^2}{a^2}$.

Exercise 155 — Page 337

1. $b^2 - 4ac = 0$, the roots are equal. **2.** They are rational and unequal. **3.** They are equal. **4.** Imaginary, equal, real and rational, real and irrational. **5.** Rational. **6.** $b^2 - 4ac = 1$. **7.** $b^2 - 4ac = 109$. **8.** $b^2 - 4ac = -191$. **9.** $b^2 - 4ac = 0$. **10.** $b^2 - 4ac = (a^2 - b^2)^2$. **11.** $b^2 - 4ac = m^2 + 4$. **12.** $b^2 - 4ac = a^2 - 4b$ and is positive if b is negative. **13.** If the roots are equal $144 - 36k = 0$ or $k = 4$. **14.** $100 - 4a^2 = 0$ or $a = \pm 5$. **15.** $b^2 - 4ac = (1 - k)^2$. **16.** $4(1 + a)^2 - 4a^2 = 0$, $a = -\frac{1}{2}$.

17. $x = 2 \pm \sqrt{k - 1}$, so that $k - 1$ cannot be negative.

18. The equation is $3x^2 + x(2a + 2b) + ab = 0$. The discriminant $= (2a + 2b)^2 - 12ab = 4(a^2 - ab + b^2)$.

19. Substitute $y = mx + c$ in $y^2 = 4ax$ and $m^2x^2 + x(2mc - 4a) + c^2 = 0$. If the roots are equal $(2mc - 4a)^2 - 4m^2c^2 = 0$.

20. $b^2 - 4ac = (5m + 2)^2 - 8m(4m + 1) = 0$ or $(m - 2)(7m + 2) = 0$.

Exercise 156 — Page 340

1. $(x - 5)(3x - 2)$. **2.** $(4x - 9)(5x + 12)$. **3.** $(x + 41)(x - 43)$. **4.** $(40a - 1)(45a + 1)$. **5.** $(13x + 1)(23x - 1)$. **6.** $(13x - 17a)(17x - 13a)$. **7.** $b^2 - 4ac = 225 - 192 = 33$ which is not a square. **8.** If $x^2 + 4x - 3 = 0$, $x = -2 \pm \sqrt{7}$. Therefore the factors of $x^2 + 4x - 3$ are $(x + 2 - \sqrt{7})(x + 2 + \sqrt{7})$. **9.** $b^2 - 4ac = 64 - 4k = 0$, then $k = 16$.

10. $b^2 - 4ac = k^2 - 36a^2$, then $k = \pm 6a$.

11. The roots of $x^2 - 6x - 11 = 0$ are $3 \pm 2\sqrt{5}$, the factors $= (x - 3 - 2\sqrt{5})(x - 3 + 2\sqrt{5})$.

12. $(9x^2 - 16y^2)(16x^2 - 9y^2) = (3x + 4y)(3x - 4y)(4x + 3y)(4x - 3y)$.

13. If $x - 2$ is a factor then the expression must vanish when $x = 2$, then $960 - 668 - 2a + 56 = 0$ or $a = 174$. Now divide by $x - 2$ and factor the quotient.

14. When the square root is taken in the usual way the root is $x\sqrt{a} + \dfrac{b}{2\sqrt{a}}$ and the remainder is $c - \dfrac{b^2}{4a}$. If it is a perfect square this remainder is zero.

Exercise 157 — Page 341

1. $-\dfrac{b}{a}$, $\dfrac{c}{a}$. **2.** $a = c$, $b = 0$. **3.** $b^2 = 4ac$, $b^2 - 4ac$ is positive, $b^2 - 4ac$ is negative, $b^2 - 4ac$ is a perfect square.

5. $2x^2 - 5x + 1 = 0$, sum $= 2\frac{1}{2}$, product $= \frac{1}{2}$.

6. The roots $= \frac{7}{2}, \frac{3}{2}$. The new roots $= \frac{7}{4}, \frac{3}{4}$, then $x^2 - \frac{5}{2}x + \frac{21}{16} = 0$.

7. $m + n = \frac{11}{3}$, $mn = \frac{1}{3}$, then $m^2 + n^2 = (m+n)^2 - 2\,mn = 12\frac{7}{9}$.

8. The roots $= \frac{3}{4}, \frac{5}{6}$. The new roots $= \frac{3}{2}, \frac{5}{3}$, then $x^2 - \frac{19}{6}x + \frac{5}{2} = 0$.

9. $b^2 - 4\,ac = 100 + 4\,k = 0$, then $k = -25$.

10. $(m+n)^2 = 144$ or $m + n = \pm 12$, then $x^2 \pm 12\,x + 35 = 0$.

11. If $5256\,x^2 + x - 1 = 0$, $x = \frac{1}{73}$ or $-\frac{1}{72}$, then $(73\,x - 1)(72\,x + 1) = 0$.
If $221\,x^2 - 8\,x - 165 = 0$, $x = \frac{15}{17}$ or $-\frac{11}{13}$, then $(17\,x - 15)(13\,x + 11) = 0$.

12. $m^3 + n^3 = (m+n)^3 - 3\,mn(m+n)$. Since $m^3 + n^3 = 28$ and $m + n = 4$, $mn = 3$. The equation is $x^2 - 4\,x + 3 = 0$.

13. $x^2 - x(2\,a + 2\,b - 2\,c) + a^2 + b^2 - c^2 = 0$. Sum $= 2\,a + 2\,b - 2\,c$.

14. $m + n = -\dfrac{53}{17}$, $mn = -\dfrac{97}{17}$. Sum of new roots $\dfrac{1}{m} + \dfrac{1}{n} = \dfrac{m+n}{mn}$
$= \dfrac{53}{97}$. Prod. $= \dfrac{1}{mn} = -\dfrac{17}{97}$, then $x^2 - \dfrac{53}{97}x - \dfrac{17}{97} = 0$.

15. If $x^2 + 6\,x + 7 = 0$, $x = -3 \pm \sqrt{2}$, then $(x + 3 - \sqrt{2})(x + 3 + \sqrt{2}) = 0$.

16. The roots $= \frac{3}{2}, -7$. The new root $= \frac{17}{2}, 0$, then $x(2\,x - 17) = 0$.

17. $m + n = -\dfrac{b}{a}$, $mn = \dfrac{c}{a}$, then $3\,m + 3\,n = -\dfrac{3\,b}{a}$, $9\,mn = \dfrac{9\,c}{a}$.

18. $\dfrac{m}{n} + \dfrac{n}{m} = \dfrac{(m+n)^2 - 2\,mn}{mn} = \dfrac{b^2 - 2\,ac}{ac}$ and $\dfrac{m}{n} \times \dfrac{n}{m} = 1$. The new equation is $x^2 - x\,\dfrac{b^2 - 2\,ac}{ac} + 1 = 0$.

19. The discriminant $= 4(a+b)^2 - 4(a + b + c)(a + b - c) = c^2$.

20. Let them be m and $2\,m$, then $3\,m = p$, $2\,m^2 = q$ or $2\,p^2 = 9\,q$.

21. $m + n = -p$, $mn = q$, then $m + n - mn = -p - q$ and $-mn(m+n) = pq$. The second equation then is $x^2 - x(p + q) + pq = 0$, whose roots are p, q.

22. The equation is $m^4x^2 - 2\,am^2x + a^2 = 0$. Since $b^2 - 4\,ac = 4\,a^2m^4 - 4\,a^2m^4 = 0$, the roots are equal.

23. $b^2 - 4\,ac = (2 + k)^2 - 4(k + 37) = 0$ or $k^2 = 144$, $k = \pm 12$.

24. When $x = 10$ or -2; for all values between 10 and -2; for all values greater than 10 or less than -2.

25. If the parts are x, $6 - x$ then $x(6 - x) = 10$ or $x^2 - 6\,x + 10 = 0$. Since $b^2 - 4\,ac = -4$, the roots are imaginary.

26. $b^2 - 4\,ac = (k + 8)^2 - 16(k + 5) = 0$, or $k^2 - 16 = 0$, $k = \pm 4$.

27. If the roots are a and b, then $a + b = -m$, $ab = n$, then $a^3 + b^3 = (a+b)^3 - 3\,ab(a + b) = -m^3 + 3\,mn$.

28. $m + n = 1 + a$, $mn = \frac{1}{2}(1 + a + a^2)$, then $m^2 + n^2 = (m + n)^2 - 2\,mn = (1 + a)^2 - (1 + a + a^2) = a$.

29. The equation reduces to $x^2(a + b - c) - 2\,abx + abc = 0$.

30. Sum $= \frac{6}{a}$, product $= 12$. If $\frac{6}{a} = 12$, $a = \frac{1}{2}$.

31. $x^2 - x(b + c) - a^2 + ac + ab = 0$. The sum of the roots is $b + c$, therefore if one root is a, the other is $b + c - a$.

32. Their difference is $6(x - a)$, therefore if they have a common factor it must be $x - a$ and $x = a$ must be the common root. If $x = a$ is a root of $x^2 - 5\,x - 3\,a = 0$, $a^2 - 5\,a - 3\,a = 0$ and $a = 8$ or 0.

33. The sum of the roots of the correct equation was 9 and their product was 8. The equation therefore was meant to be $x^2 - 9\,x + 8 = 0$.

34. $m + n = -\frac{b}{a}$, $mn = \frac{c}{a}$. Then $m + n + \frac{1}{m} + \frac{1}{n} = -\frac{b(a + c)}{ac}$ and $\left(m + n\right)\left(\frac{1}{m} + \frac{1}{n}\right) = \frac{b^2}{ac}$, and these are the correct sum and product of the roots of the second equation.

35. The correct sum of the roots is 8 and the correct product is 12. Therefore the equation was $x^2 - 8\,x + 12 = 0$.

36. If $x^2 + 2\,bx + c = 0$, $x = -b \pm \sqrt{b^2 - ac}$. Therefore the factors are $(x + b + \sqrt{b^2 - ac})(x + b - \sqrt{b - ac})$.

Exercise 158 — Page 345

10. $(a - 2\,b - 4)(a - 2\,b + 3)$.

11. $(x + 2\,y - z)(2\,x - 3\,y + 4\,z)$.

12. $(a - 2\,b - 3\,c)(2\,a - 3\,b + c)$.

13. $(x + 2\,y - 3)(2\,x - 11\,y + 1)$.

14. $(2\,a + 3\,b - 4)(3\,a - 4\,b + 5)$.

15. $(x - 3\,z)(x + 2\,z) - 2\,y(x - 3\,z) = (x - 3\,z)(x + 2\,z - 2\,y)$.

16. $(2\,a - b + 3)(3\,a - b + 1)(a + b - 2) \div (a + b - 2)(3\,a - b + 1)$.

17. The numerator $= (p - 7\,q + 3\,r)(4\,p - 3\,q - 6\,r)$. The denominator $= (p - q + r)(4\,p - 3\,q - 6\,r)$.

18. From the terms $3\,x^2$, $-6\,y^2$, $-10\,z^2$ it is seen that the other factor must be $x - 3\,y + 2\,z$. Now find the product of the two factors and the values of a, b, c are evident.

19. $$x = \frac{1 - y \pm \sqrt{(1 - y)^2 + 4(2\,y^2 - 10\,y + 12)}}{2} = \frac{1 - y \pm (3\,y - 7)}{2}$$

$= y - 3$ or $-2\,y + 4$. The factors $= (x - y + 3)(x + 2\,y - 4)$.

20. (1) The factors $= (x - 2\,a + 3)(x - 3\,a + 4)$, then $x = 2\,a - 3$ or $3\,a - 4$. (2) Arrange as a quadratic in x and solve as in Ex. 19.

21. The factors $= (2\,a - b + c)(3\,a - b - c)$ and $(2\,a - b + c)(3\,a + 2\,b - 2\,c)$.

Exercise 159 — Page 347

1. $(a + 2b - c)\{(a + 2b)^2 + c(a + 2b) + c^2\}$.

2. $(a - b + c)\{a^2 + a(b - c) + (b - c)^2\}$.

3. $(a + b + 2c)\{(a + b)^2 - 2c(a + b) + 4c^2\}$.

4. $(a + b + c + d)\{(a + b)^2 - (a + b)(c + d) + (c + d)^2\}$.

5. $(x - y - a + b)\{(x - y)^2 + (x - y)(a - b) + (a - b)^2\}$.

6. $(2x - y + x - 2y)\{(2x - y)^2 - (2x - y)(x - 2y) + (x - 2y)^2\} = 9(x - y)(x^2 - xy + y^2)$. **7.** $2(a + b)(13a^2 - 22ab + 13b^2)$.

8. $(6a - 2b)^3 - (6a - 9b)^3 = (6a - 2b - 6a + 9b)\{(6a - 2b)^2 + (6a - 2b)(6a - 9b) + (6a - 9b)^2\} = 7b(108a^2 - 198ab + 103b^2)$.

9. $(a - b + c)(a^2 + b^2 + c^2 + ab - ac + bc)$.

10. $(2x + y + z)(4x^2 + y^2 + z^2 - 2xy - 2xz - yz)$.

11. $(a + b - 1)(a^2 + b^2 + 1 - ab + a + b)$.

12. $(1 + c - d)(1 + c^2 + d^2 - c + d + cd)$.

13. This is of the form $a^3 + b^3 + c^3 - 3abc$ where $a = 2x$, $b = -y$. $c = -5z$. (See Art. 245.)

22. This is the difference of two cubes and therefore one factor is $(4a + 3b) - (a + 2b)$ or $3a + b$.

23. $(x^2 - 3x + 7) + 2$ or $x^2 - 3x + 9$. **24.** $(a^2 - 3a + 2) - (a^2 - 5a + 7)$.

26. It is divisible by $(4a^2 + a + 1) - (2a^2 - 2a + 3) = (2a - 1)(a + 2)$.

29. One factor of $a^3 + b^3 - c^3 + 3abc$ is $a + b - c$, but $a + b = c$ or $a + b - c = 0$, therefore $a^3 + b^3 - c^3 + 3abc = 0$ or $a^3 + b^3 + 3abc = c^3$.

30. $x^3 - y^3 - z^3 - 3xyz$ is divisible by $x - y - z$ which equals zero.

31. $(a + 2b)^3 + (b + 2c)^3 + (c + 2a)^3 - 3(a + 2b)(b + 2c)(c + 2a)$ is divisible by $a + 2b + b + 2c + c + 2a$ or $3(a + b + c)$ which equals zero.

32. One factor is $x - y + y - z + z - x$ which equals zero.

33. It follows since $(a + 3b - 4c) + (b + 3c - 4a) + (c + 3a - 4b)$ equals zero.

34. $x + y - z = 0$, then $x^3 + y^3 - z^3 + 3xyz = 0$.

35. $a - b + c = 0$, then $a^3 - b^3 + c^3 + 3abc = 0$.

36. A common factor of the numerator and denominator of the first is $a - b - c$ and of the second $x^2 + 4y^2 + z^2 - 2xy - xz - 2yz$.

37. One factor $= ax + by + az + bx + ay + bz = (a + b)(x + y + z)$.

38. $x + y + z = 2(a + b + c)$. Remove the factor $x + y + z$ from the first side and $2(a + b + c)$ from the second and the remaining factors may be shown equal by substituting the values of x, y, z.

39. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}$. It follows from this identity because if a and b are increased or decreased by the same quantity it does not alter the value of $a-b$.

40. Since $x-a+b-x+a-b=0$, it follows that $3(x-a)(b-x)(a-b)=0$ or $x=a, b$. (See Art. 245, Ex. 4.)

Exercise 160 — Page 350

Examples **1–8** are factored by cross multiplication. See Art. 246, Ex. 4.

1. $(ax+b)(cx+d)$. **2.** $(mx-ny)(px+qy)$.

3. $\{x(a-b)+a+b\}\{x(a+b)-(a-b)\}$.

4. $\{y(p+q)+p-q\}\{y(p-q)+p+q\}$.

5. $(x+1)(ax+bx+b+c)$. **6.** $(ax+a-2)\{(a-1)x+a\}$.

7. $\{a(b+c)+bc\}\{a+(b+c)\}$.

8. Arrange in descending powers of a and it is the same as Ex. 7.

9–14. The factors in each case may be found by arranging the expressions in descending powers of a selected letter as in Art. 246, Exs. 1, 2.

15. $(ax-c)(bx-d)=0$.

16. $\{(a-b)x-(a+b)\}\{(a+b)x+(a-b)\}=0$.

17. $x^2(a-b)-x(a^2-b^2)+ab(a-b)=0$ or $x^2-x(a+b)+ab=0$.

18. $\{ax-(a+b)\}\{bx-(a-b)\}=0$.

19. $(ax-b)\{(a-b)x+(a+b)\}=0$.

20. $(1)=\{ax-(a+b)\}\{bx+(a-b)\}$, $(2)=(ax+b)\{ax-(a+b)\}$.

Exercise 161 — Page 353

1. $f(1)=1-8+19-12=0$, $f(2)=8-32+38-12=2$, $f(3)=27-72+57-12=0$, $f(4)=64-128+76-12=0$, $f(5)=125-200+95-12=8$. The factors $=(x-1)(x-3)(x-4)$.

2. 0, 0, 0, 0, 24. The factors $=x(x-2)(x-1)(x+1)$.

3. It vanishes when $x=y$ and when $x=-y$.

4. It vanishes when $x=\pm 1$, when $x^2=-1$, when $x^3=-1$.

5. $x^{17}+y^{17}$ vanishes when $x=-y$, x^5+32 vanishes when $x=-2$.

6. x^2-xy+y^2. **7.** $a^3+a^2b+ab^2+b^3$.

8. $a^3-a^2b+ab^2-b^3$. **9.** $x^6+x^5+x^4+x^3+x^2+x+1$.

18. When $x^2=-7$, $f(x)=-343+147-28+224=0$.

20. When a is substituted for x, $a^2+pa+q=0$.

21. $f(x)-f(a)=mx^2+nx+r-(ma^2+na+r)=m(x^2-a^2)+n(x-a)$.

22. When $x=1, f(x)=1-k^2+10k-10=0$ or $k^2-10k+9=0$.

Exercise 162 — Page 357

1. a, b. **2.** a, c. **3.** x, y. **4.** a, b, c. **5.** a, b, c. **6.** None. **7.** p, q, r. **8.** $x+y$, $a+b+c$, $a+b+c+d$. **9.** $a^2+b^2+c^2+3ab+3ac+3bc$. **10.** $2(a^2+b^2+c^2+ab+ac+bc)$; $2(a^2+b^2+c^2-ab-bc-ca)$. **11.** $(b+c)(c+a)$.

12. $2(a^3+b^3+c^3)+3(a^2b+b^2c+c^2a+ab^2+bc^2+ca^2)$.

13. The coefficient of a^2 is 3 and therefore also of b^2 and c^2. The coefficient of ab is -2 and therefore also for bc and ca since the expression is symmetrical.

14. The coefficient of a^2 is 2 and of ab is 0. (See Art. 252, Ex. 4.)

15. The coefficients of x^2, y^2, z^2 are 0 and also of xy, yz, zx.

16. In the result there may be terms like a^3, a^2b, and abc. The coefficient of a^3 is zero and there is no term containing abc. There are therefore only terms like a^2b and these evidently are $-3a^2b+3ab^2$, etc.

17–18. Put $x=y$ and it vanishes, therefore $x-y$ is a factor and so $y-z$, $z-x$ are factors. Now proceed as in Art. 252, Ex. 1, to show that the numerical factor is -1.

19. Put $a=-b$ and it vanishes so that $a+b$, $b+c$, $c+a$ are factors. Now find the numerical factor, which is 1.

20. See Art. 252, Ex. 2. **21.** Put $x=y$, and $x-y$, $y-z$, $z-x$ are factors. The numerical factor is 3.

22. When simplified this is the same as Ex. 19.

23. This expression is factored in Art. 252, Ex. 3.

24. This is the same as $x(y^2-z^2)+$ etc., where $x=a^2$, $y=b^2$, $z=c^2$.

25. The numerator is $x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)$, which equals the denominator and the result is 1.

26. $$\frac{-x^2(y-z)-y^2(z-x)-z^2(x-y)}{(x-y)(y-z)(z-x)}=\frac{(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)}=1.$$

27. $$\frac{-ab(a-b)-bc(b-c)-ca(c-a)}{(a-b)(b-c)(c-a)}=\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=1.$$

28. $$\frac{a^2(b-c)+b^2(c-a)+c^2(a-b)}{abc(a-b)(b-c)(c-a)}=\frac{-(a-b)(b-c)(c-a)}{abc(a-b)(b-c)(c-a)}=\frac{-1}{abc}.$$

29. $$\frac{(b^2-ac)(c-a)+(c^2-ba)(a-b)+(a^2-cb)(b-c)}{(a-b)(b-c)(c-a)}.$$ When the numerator is simplified, the terms all cancel.

30. $\dfrac{-bc(b^2-c^2)-ca(c^2-a^2)-ab(a^2-b^2)}{(a-b)(b-c)(c-a)} = a+b+c$, since the factors of the numerator $= (a-b)(b-c)(c-a)(a+b+c)$. (See Art. 252, Ex. 3.)

31. $\dfrac{x^3(y-z)+y^3(z-x)+z^3(x-y)}{(x-y)(y-z)(z-x)} = -(x+y+z)$.

32. $\dfrac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$.

33. Since a is a factor, b and c are factors. The remaining factor must be a numerical one. Put $a=b=c=1$ and the numerical factor is 6. **34.** $b-c$ and $c-a$ are also factors.

35. Let the expression $= a(x^2+y^2+z^2)+b(xy+yz+zx)$. When $x=y=z=1$, $3a+3b=15$; when $x=1$, $y=2$, $z=3$, $14a+11b=64$, from which $a=3$, $b=2$.

36. In each case the expression on the left is symmetrical and therefore those on the right must be also. In (1) if a^3 occurs then b^3 and c^3 must also, and if $a^2(b+c)$ occurs then $b^2(c+a)$ and $c^2(a+b)$ must also. In (3) if ab^2 occurs then bc^2 and ca^2 must also appear with the same signs. In (2) the expression on the right is symmetrical, but if multiplied out it would evidently contain terms like a^2b, which do not appear on the left.

Exercise 163 — Page 360

1–3. Perform the multiplications and simplify, and the results follow.

4. This follows from the factors of $a^3+b^3+c^3-3abc$.

5. Factor the first side as in Art. 252, Ex. 3.

6. One factor is $(3a-2b+4c)-(2a-3b+3c)=a+b+c$.

7. $(a+b)^2-c^2=(a+b+c)(a+b-c)=0$; $c^2-ab-b^2+ac= (c+b)(c-b)+a(c-b)=(c-b)(a+b+c)=0$.

8. For $a+b$, $b+c$, $c+a$ write $-c$, $-a$, $-b$ respectively.

9. $a^4+b^4+c^4-2a^2b^2-2b^2c^2-2c^2a^2=(a^2+b^2-c^2)^2-4a^2b^2= (a^2+b^2-c^2+2ab)(a^2+b^2-c^2-2ab)=(a+b+c)(a+b-c)(a-b+c)(a-b-c)=0$, since $a+b+c=0$.

10. One factor of the difference is $(3a-b)+(3b-c)+(3c-a)= 2(a+b+c)=0$.

11. Since $b+c=-a$, $b^2+bc+c^2=a^2-bc$. Similarly, $c^2+ca+a^2 = b^2-ac$ and $a^2+ab+b^2=c^2-ab$. Now the expression $= a^3+b^3+c^3 -3abc=0$ since $a+b+c$ is a factor.

12. $(a^2-b^2)^2=(a+b)^2(a-b)^2=(a-b)^2$; $a^3+b^3-ab=(a+b)(a^2-ab+b^2)-ab=a^2-2ab+b^2=(a-b)^2$.

13. Since $x = 2z - y$, $\dfrac{x}{x-z} + \dfrac{y}{y-z} = \dfrac{2z-y}{z-y} + \dfrac{y}{y-z} = \dfrac{2z-2y}{z-y} = 2.$

14. Find the sum and product of a, b, c and the result follows.

15. $\dfrac{2a-c}{a^2-ac} = \dfrac{2}{a-b}$, $2a^2 - 2ac = 2a^2 - ac - 2ab + bc$, $2ab = ac + bc$, then $\dfrac{a+b}{ab} = \dfrac{2}{c}$, or $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{2}{c}$.

16. $\left(x + \dfrac{1}{x}\right)^2 = y^2$ or $x^2 + \dfrac{1}{x^2} = y^2 - 2$; $\left(x + \dfrac{1}{x}\right)^3 = x^3 + \dfrac{1}{x^3} + 3x + \dfrac{3}{x}$, then $y^3 = x^3 + \dfrac{1}{x^3} + 3y$, or $x^3 + \dfrac{1}{x^3} = y^3 - 3y$; $x^4 + \dfrac{1}{x^4} = \left(x^2 + \dfrac{1}{x^2}\right)^2 - 2 = (y^2-2)^2 - 2 = y^4 - 4y^2 + 2.$

17. $2s^2 - s(a+b+c) + bc = 2s^2 - 2s^2 + bc = bc.$

18. $2s^2 + s(a+b+c) - (a^2+b^2+c^2) = 4s^2 - (a^2+b^2+c^2) = (a+b+c)^2 - (a^2+b^2+c^2) = 2ab + 2bc + 2ca.$

19. The first side $= \{(s-a) + (s-b) + (s-c)\}^2 = (3s - 2s)^2 = s^2.$

20. $2as + bc = a(a+b+c) + bc = (a+b)(a+c)$. Similarly $2bs + ca = (b+a)(b+c)$, $2cs + ab = (c+a)(c+b)$.

21. $\dfrac{s(s-b)(s-c) + s(s-c)(s-a) + s(s-a)(s-b) - (s-a)(s-b)(s-c)}{s(s-a)(s-b)(s-c)}$

$= \dfrac{2s^3 - s^2(a+b+c) + abc}{s(s-a)(s-b)(s-c)} = \dfrac{2s^3 - s^2(2s) + abc}{s(s-a)(s-b)(s-c)} = \dfrac{abc}{s(s-a)(s-b)(s-c)}.$

22. $2s(2s-2a)(2s-2b)(2s-2c)$
$= (a+b+c)(b+c-a)(c+a-b)(a+b-c) = \{(b+c)^2 - a^2\}\{a^2 - (b-c)^2\}$
$= (b^2 + c^2 - a^2 + 2bc)(a^2 - b^2 - c^2 + 2bc) = 4b^2c^2 - (b^2+c^2-a^2)^2 = 2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4.$

23. Since $b + \dfrac{1}{c} = 1$, $b = 1 - \dfrac{1}{c} = \dfrac{c-1}{c}$ and $\dfrac{1}{a} = 1 - c$, then $a + \dfrac{1}{b} = \dfrac{1}{1-c} + \dfrac{c}{c-1} = \dfrac{1-c}{1-c} = 1$; $abc = \dfrac{1}{1-c} \cdot \dfrac{c-1}{c} \cdot c = -1.$

24. $a^3 + 3a + \dfrac{3}{a} + \dfrac{1}{a^3} = 27$, or $a^3 + \dfrac{1}{a^3} = 27 - 3\left(a + \dfrac{1}{a}\right) = 18.$

25. $\dfrac{ab}{(b+c)(c+a)} + \dfrac{bc}{(c+a)(a+b)} + \dfrac{ca}{(a+b)(b+c)} + \dfrac{2abc}{(a+b)(b+c)(c+a)}$

$= \dfrac{a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2abc}{(a+b)(b+c)(c+a)} = 1.$

26. $x^2 + y^2 = (x+y)^2 - 2xy = a^2 - 2b^2$; $x^3 + y^3 = (x+y)^3 - 3xy(x+y) = a^3 - 3ab^2.$

27. This follows at once from the preceding example.

28. $(x+y)^2 = x^2+y^2+2xy$, then $xy = \frac{1}{2}(a^2-b^2)$. $x^3+y^3=(x+y)^3 - 3xy(x+y)$, then $c^3 = a^3 - \frac{3}{2}a(a^2-b^2)$.

29. $x+y+z=a+b+c$, it is then required to prove $x^2+y^2+z^2 - xy - yz - zx = 4(a^2+b^2+c^2-ab-bc-ca)$ or $2x^2+2y^2+2z^2 - 2xy - 2yz - 2zx = 4(2a^2+2b^2+2c^2-2ab-2bc-2ca)$ or $(x-y)^2 + (y-z)^2 + (z-x)^2 = 4\{(a-b)^2+(b-c)^2+(c-a)^2\}$. This is seen to be true since $x-y=2(a-c)$, etc.

Exercise 164 — Page 362

1. See Art. 245, Ex. 5. **2.** On multiplying the result follows. **3.** For $a+c$ put $-(b+d)$, for $a+d$ put $-(b+c)$, and the result follows. **4.** Put $a=b$ and we get $(b-c)^n+(c-b)^n=(b-c)^n-(b-c)^n=0$, since n is an odd integer. Therefore $a-b$ is a factor.

5. x^n-1 is divisible by $x-1$ so 12^n-1 is divisible by $12-1$.
$x^{2n+1}+1$ is divisible by $x+1$ or by $23+1$ or 24 when $x=23$.
$x^{2n}-1$ is divisible by x^2-1 or by 7^2-1 or 48 when $x=7$.

7. When $a=0$, $a=x$, $a=2x$ the expression vanishes.

8. When $x=0$, $y=0$, $x=-y$ the expression vanishes.

9. $y-a-ay+a^2=y-b-by+b^2$ or $y(a-b)=a^2-b^2-a+b$. Then $y=a+b-1$ or $y-b=a-1$ whence $(a-1)(1-b)=x$.

10. (1) Since $x=-y-z$, $x^2+xy+y^2=(-y-z)^2-y(y+z)+y^2 = y^2+yz+z^2$. Similarly $z^2+zx+x^2=y^2+yz+z^2$.

(2) Since $x+y-z=-2z$, $x-y+z=-2y$, $y+z-x=-2x$, the expression $=(-2z)^3+(-2x)^3+(-2y)^3+24xyz=-8(x^3+y^3+z^3-3xyz) = 0$, since $x+y+z$ is a factor.

11. $$\frac{-a^2(b-c)-b^2(c-a)-c^2(a-b)}{abc(a-b)(b-c)(c-a)}=\frac{(a-b)(b-c)(c-a)}{abc(a-b)(b-c)(c-a)}=\frac{1}{abc}.$$

12. $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$ is divisible by $(x-a)+(x-b)+(x-c)$ so that $3x=a+b+c$.

13. $(a+b)^5-a^5-b^5=5a^4b+10a^3b^2+10a^2b^3+5ab^4$
$=5ab(a^3+2a^2b+2ab^2+b^3)=5ab(a+b)(a^2+ab+b^2)$.

14. (1) $2s^2-s(a+b+c)+ac=2s^2-2s^2+ac=ac$.

(2) $4s^2-2s(a+b+c)+ab+bc+ca=ab+bc+ca$.

(3) $(s-a)^3+(s-c)^3-b^3+3b(s-a)(s-c)=0$, since it is divisible by $s-a+s-c-b$ which $=0$, since $2s=a+b+c$.

15. The other factor of $a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)$ must be of the form $m(a^2+b^2+c^2)+n(ab+bc+ca)$. If $a=1$, $b=2$, $c=-1$ we get $6m-n=1$. If $a=1$, $b=2$, $c=3$ we get $14m+11n=-11$ from which $m=0$, $n=-1$, so the other factor is $-(ab+bc+ca)$. (See Art. 252, Ex. 3.)

16. $\dfrac{-bc(x-a)(b-c)-ca(x-b)(c-a)-ab(x-c)(a-b)}{abc(a-b)(b-c)(c-a)}$
$=\dfrac{x(bc^2-b^2c+ca^2-c^2a+ab^2-a^2b)}{abc(a-b)(b-c)(c-a)}=\dfrac{x}{abc}.$

17. Substitute for x, y, z and it follows from the factors of $a^3+b^3+c^3-3abc$.

18. The numerator $=-(a-b)(b-c)(c-a)(a+b+c)$. The denominator $=3(a-b)(b-c)(c-a)$.

19. (1) This follows at once on squaring $a+b+c$. (2) Divide each side by $a+b+c$, then it is required to prove $(a+b+c)^2=a^2+b^2+c^2-ab-bc-ca$. (3) From (1), $(a+b+c)^4=(a^2+b^2+c^2)^2=a^4+b^4+c^4+2a^2b^2+2b^2c^2+2c^2a^2$. It is now required to show that $2a^2b^2+2b^2c^2+2c^2a^2=-4abc(a+b+c)$ or that $a^2b^2+b^2c^2+c^2a^2+2abc(a+b+c)=0$, which is true since it $=(ab+bc+ca)^2=0$.

20. $x^n(x-1)-(x-1)=(x-1)(x^n-1)$, which is divisible by $(x-1)^2$ since x^n-1 is divisible by $x-1$.

21. $x^3+x^2a+xa^2+a^3$, x^4-x^2+1, $a^4+2a^3+4a^2+8a+16$.

22. Divide by $x-1$, which is evidently a factor of each term.

23. $\dfrac{-a(b^2+bc+c^2)(b-c)\cdots}{(a-b)(b-c)(c-a)}=\dfrac{-a(b^3-c^3)\cdots}{(a-b)(b-c)(c-a)}$
$=\dfrac{-(a-b)(b-c)(c-a)(a+b+c)}{(a-b)(b-c)(c-a)}=-(a+b+c).$

24. The expression $=(x-y)(y-z)(z-x)$. The change of x to $x+a$, etc., will not alter these factors.

25. $x^4=x^2+2x+1=x+1+2x+1=3x+2$. Then $x^5=x(3x+2)=3x^2+2x=3(x+1)+2x=5x+3$.

26. A factor $=ax+b+bx+c+cx+a=(x+1)(a+b+c)$.

27. Since $a+b-c$ is a factor of $a^3+b^3-c^3+3abc$, then $x^{\frac{2}{3}}+y^{\frac{2}{3}}-z^{\frac{2}{3}}$ is a factor of $x^2+y^2-z^2+3x^{\frac{2}{3}}y^{\frac{2}{3}}z^{\frac{2}{3}}$. Therefore $x^2+y^2-z^2=-x^{\frac{2}{3}}y^{\frac{2}{3}}z^{\frac{2}{3}}$ or cubing each side $(x^2+y^2-z^2)^3=-27x^2y^2z^2$.

28. For $a+b$ put $-c$ etc., then we have to show $a^3+b^3+c^3-3abc=0$, which is true since $a+b+c$ is a factor.

29. See Ex 35, page 358. **30.** See Ex. 27, page 362.

31. $(x+y)^2=x^2+y^2+2xy$ or $9=5+2xy$, then $xy=2$. $x^3+y^3=(x+y)^3-3xy(x+y)=27-18=9$. $x^4+y^4=(x^2+y^2)^2-2x^2y^2=25-8=17$.

32. $a^2+b^2+c^2=(a+b+c)^2-2(ab+bc+ca)=100-62=38$. $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=70$.

Exercise 165 — Page 367

1–12. In $a + (n - 1)d$ substitute the particular values for a and d and for the 5th and 12th terms put $n = 5$ and 12 respectively. Thus in Ex. 1, $a = 3$, $d = 5$; in Ex. 2, $a = 11$, $d = -5$; in Ex. 3, $a = \frac{1}{2}$, $d = 1$.

13. $1 + (n - 1)1 = n$.

14. $1 + (n - 1)2 = 2n - 1$.

15. $7 + (n - 1)3 = 3n + 4$.

16. $2 + (n - 1)(-3) = 5 - 3n$.

17. $3x + (n - 1)(-2x)$.

18. $n - 1 + (n - 1)(-1) = 0$.

19. $(a + b)^2 + (n - 1)(-2ab)$.

20. $2 - \frac{1}{n} + (n - 1)\left(-\frac{1}{n}\right) = 1$.

21. $1 + 2(n - 1) = 39$, $n = 20$.

22. $9 - 3(n - 1) = -288$, $n = 100$.

23. $21 - 1\frac{1}{2}(n - 1) = 0$, $n = 15$.

24. $\frac{2}{3} - \frac{1}{6}(n - 1) = -3$, $n = 23$.

25. $3 + 19d = 79$, $d = 4$.

26. $a + 7d = 50$, $a + 20d = 115$, $a = 15$, $d = 5$.

27. $d = 3$, then $2 + 3(n - 1) = 119$, whence $n = 40$.

28. $a + 14d = -40$, $a + 30d = 40$; $a = -110$, $d = 5$.

29. $a + 2d = 9$, $a + 9d = -12$; $a = 15$, $d = -3$.
Then $15 - 3(n - 1) = -60$.

30. Suppose the nth term is 193, then $13 + 2(n - 1) = 193$, $n = 91$.

31. The first is 105 and the last is 497. If n is the number then $105 + 7(n - 1) = 497$, whence $n = 57$.

32. $a + d + a + 4d = 26$, $a + 3d + a + 19d = 94$, $a = 3$, $d = 4$.

33. Sum of the first three $= 3a + 3d$, of the next three $= 3a + 12d$.

34. $199 - 4 \times 19 = 123$.

35. $62 - 3 \times 7 + 2 + 3 \times 7 = 64$.

36. rth from the beginning $= a + d(r - 1)$, rth from the end $= l - d(r - 1)$. The sum $= a + l$, which is constant.

37. Let them be $a - d$, a, $a + d$, then $3a = 30$, $a = 10$.

38. $3a = 15$ or $a = 5$; $(5 - d)^2 + 5^2 + (5 + d)^2 = 83$, $d = 2$.

39. $a + 9d = 29$, $a + 19d = 59$, $a + (n - 1)d = 137$; $a = 2$, $d = 3$, $n = 46$.

40. $(2x - 2y) - (x + y) = (5x - 6y) - (2x - 2y)$, whence $y = 2x$.

41. Let them be $a - d$, a, $a + d$, then $(a + d)^2 = a^2 + (a - d)^2$ or $a^2 - 4ad = 0$ or $a = 4d$. The sides are $3d$, $4d$, $5d$.

42. The $(n - r + 1)$th term $= a + (n - r)d$, $(n + 1)$th $= a + nd$, $(n + r - 1)$th $= a + (n + r)d$. These are in A. P. as the difference between each consecutive pair is rd.

Exercise 166 — Page 370

1-6. Use the formula $S = \frac{n}{2}(a + l)$ and substitute the given values of a, l and n.

7-16. Use the formula $S = \frac{n}{2}(2a + \overline{n-1}\,d)$ and substitute the values of a, d and n.

17. $S = \frac{n}{2}\{4 + 3(n-1)\} = \frac{n}{2}(3n + 1)$. **18.** $\frac{n}{2}(2 + n - 1)$.

19. $\frac{n}{2}\{10 + 6(n-1)\}$. **20.** $\frac{n}{2}\{6 - 2(n-1)\}$. **21.** $\frac{n}{2}\{8 - 2\frac{1}{2}(n-1)\}$.

22. $\frac{n}{2}\{2a + 4a(n-1)\}$. **23.** $\frac{n}{2}\{2(a+1) + 2(n-1)\}$.

24. $\frac{n}{2}\{2\left(1 - \frac{1}{n}\right) - \frac{1}{n}(n-1)\}$. **25.** $\frac{n}{2}\{2 + 2(n-1)\} = n^2$.

26. $S = 125 + 123 + 121 \cdots$ to 20 terms $= 10\{250 - 2 \times 19\} = 2120$.

27. $\frac{n}{2}\{32 - 2(n-1)\} = 72$ or $n^2 - 17n + 72 = 0$; $n = 8$ or 9.

28. $41\frac{1}{4} = \frac{n}{2}\{9 - \frac{1}{4}(n-1)\}$, $n^2 - 37n + 330 = 0$, $n = 15$ or 22.

29. $68 = \frac{n}{2}\{6 + \frac{1}{8}(n-1)\}$, $n^2 + 47n - 1088 = 0$, $n = 17$ or -64.

30. $a + 4d = 8$, $a + 9d = -2$, then $a = 16$, $d = -2$ and the sum of 17 terms $= \frac{17}{2}\{32 - 2 \times 16\} = 0$.

31. $a + 3d + a + 7d = 24$, $a + 14d + a + 18d = 68$, from which $a = 2$, $d = 2$. Sum of n terms $= \frac{n}{2}(4 + 2n - 2)$.

32. Sum of 10 terms $= \frac{10}{2}(2a + 9d) = 10a + 45d$. Sum of the next 10 terms $= \frac{10}{2}(2a + 20d + 9d) = 10a + 145d$. If $10a + 45d = 100$ and $10a + 145d = 300$, $a = 1$, $d = 2$.

33. Suppose A overtakes B in x hours, then B has travelled $4x$ miles and the number of miles A has travelled $= 2 + 2\frac{1}{4} + 2\frac{1}{2} + \cdots$ to x terms $= \frac{x}{2}\{4 + \frac{1}{4}(x-1)\}$. $\therefore \frac{1}{2}x(3\frac{3}{4} + \frac{1}{4}x) = 4x$, whence $x = 17$.

34. The distance in yards $= 2 + 4 + 6 + \cdots$ to 39 terms.

35. The middle term is the $(n+1)$th term $= a + nd$. Sum of $2n + 1$ terms $= \frac{2n+1}{2}(2a + 2nd) = (2n+1)(a + nd)$.

36. The first term is 1002 and the last is 1998. If there are n terms, then $1002 + 6(n-1) = 1998$ or $n = 167$. The sum of 167 terms whose first term is 1002 and last $1998 = \frac{167}{2}(1002 + 1998)$.

37. In all there are 91 integers whose sum $= \frac{91}{2}(10 + 100) = 5005$. Of these 30 are divisible by 3 and their sum $= 12 + 15 + \cdots$ to 30 terms $= 1665$. Then the sum of those not divisible $= 3340$.

38. The height $= 16.1 + 48.3 + 80.5 + \cdots$ to 12 terms

$$= 6(32.2 + 11 \times 32.2).$$

39. $41 \times \frac{n}{2}\{2 + 3(n-1)\} = 10 \times \frac{2n}{2}\{2 + 3(2n-1)\}$, whence $n = 7$.

40. $a + d = 28$, $a + 5d = 12$, $a + (n-1)d = -28$, then $a = 32$, $d = -4$, $n = 16$. Sum of 16 terms $= 8(32 - 28) = 32$.

41. Suppose a is the required term, then since $d = -2$, $s = 960$, $n = 40$, $960 = 20(2a - 78)$, whence $a = 63$.

42. Put $n = 1$ and the sum of 1 term or the first term $= -1$. Put $n = 2$ and the sum of 2 terms $= 2$. Then the second term must be 3 and the series $= -1, 3, 7 \cdots$.

Exercise 167 — Page 372

1. $18, \quad -26, \; a, \; 4x + y$.

2. Since 53 is the 8th term of which 11 is the first, then $11 + 7d = 53$ or $d = 6$ and therefore the means are 17, 23, 29, 35, 41, 47. If 12 means are inserted between $4a + 3b$ and $3b - 9a$, then $4a + 3b + 13d = 3b - 9a$ or $d = -a$ and the means are $4a + 2b$, $4a + b$, $4a$, $4a - b$, etc.

3. $65 = 2 + 21d$ or $d = 3$. Then $S = \frac{20}{2}(5 + 62) = 670$.

4. The 102nd term $= 493$, then $493 = -12 + 101d$ or $d = 5$. The 65th mean $= -7 + 64 \times 5 = 313$.

5. $y = x + (m+1)d$ or $d = \dfrac{y - x}{m + 1}$. The rth mean $= x + \dfrac{r(y - x)}{m + 1}$.

6. $\frac{1}{2}(ax^2 + bx + ax + bx^2) = a + b$ or $\frac{1}{2}(a + b)(x^2 + x) = a + b$. Then $x^2 + x = 2$ or $x = 1$ or -2.

7. $b = \dfrac{a + c}{2}$, $x = \dfrac{1}{2}\left(a + \dfrac{a + c}{2}\right) = \dfrac{3a + c}{4}$, $y = \dfrac{1}{2}\left(\dfrac{a + c}{2} + c\right) = \dfrac{a + 3c}{4}$.

We now wish to show that a, $\dfrac{3a + c}{4}$, $\dfrac{a + c}{2}$, $\dfrac{a + 3c}{4}$, c are in A. P. They are in A. P. since they have a common difference which is $\frac{1}{4}(c - a)$.

8. Since b is the 4th term $a + 3d = b$ or $d = \frac{1}{3}(b - a)$. Then $x = a + d = a + \frac{1}{3}(b - a)$ and $y = b - d = b - \frac{1}{3}(b - a)$.

Exercise 168 — Page 375

1. $\frac{1}{60}, \frac{1}{30}, \frac{1}{20}$ are in A. P. and $d = \frac{1}{60}$. The next two terms of the A. P. are $\frac{1}{20} + \frac{1}{60}$ and $\frac{1}{20} + \frac{2}{60}$ or $\frac{1}{15}$ and $\frac{1}{12}$. Then the next two terms of the H. P. are 15 and 12.

5. $\frac{1}{6}, -\frac{1}{12}, -\frac{1}{3}$ are in A. P. and $d=-\frac{1}{4}$. The next two terms of the A. P. are $-\frac{1}{3}-\frac{1}{4}$ and $-\frac{1}{3}-\frac{1}{2}$ or $-\frac{7}{12}$ and $-\frac{5}{6}$. Then the next two terms of the H. P. are $-\frac{12}{7}$ and $-\frac{6}{5}$.

6. $\sqrt{2}, \dfrac{1}{\sqrt{2}-1}, \dfrac{2}{2-\sqrt{2}}$ or $\sqrt{2}, \sqrt{2}+1, \sqrt{2}+2$ are in A. P. and $d=1$. The next two terms of the A. P. are $\sqrt{2}+3$ and $\sqrt{2}+4$. Then the next two terms of the H. P. are $\dfrac{1}{\sqrt{2}+3}$ and $\dfrac{1}{\sqrt{2}+4}$.

7. In the A. P. $\frac{1}{24}, \frac{1}{12}, \frac{1}{8}, \cdots, d=\frac{1}{24}$ and the 24th term $=\frac{1}{24}+\frac{23}{24}=1$. Then the 24th term of the H. P. $=1$.

8. In the A. P. $2, \frac{7}{4}, \frac{3}{2}, \cdots, d=-\frac{1}{4}$ and the nth term $=2-\frac{1}{4}(n-1)=\dfrac{9-n}{4}$. Then the nth term of the H. P. $=\dfrac{4}{9-n}$.

9. The first two terms of the A. P. are $\dfrac{1}{a}, \dfrac{1}{b}$ and $d=\dfrac{1}{b}-\dfrac{1}{a}$, then the nth term of the A. P. $=\dfrac{1}{a}+\left(n-1\right)\left(\dfrac{1}{b}-\dfrac{1}{a}\right)$. Then the nth term of the H. P. is the reciprocal of this quantity.

10. The first two terms of the A. P. are $\frac{1}{60}, \frac{1}{30}$, and $d=\frac{1}{60}$. Then the terms of the A. P. are $\frac{1}{60}, \frac{1}{30}, \frac{1}{20}, \frac{1}{15}, \frac{1}{12}, \frac{1}{10}$. The sum of the terms of the H. P. $=60+30+20+15+12+10=147$.

11. In the A. P., $a+4d=\frac{1}{168}$ and $a+7d=\frac{1}{105}$, whence $a=\frac{1}{840}$ and $d=\frac{1}{840}$. Then the first two terms of the A. P. are $\frac{1}{840}, \frac{1}{420}$.

12. In the A. P., $a=\frac{5}{12}$ and $a+3d=-\frac{1}{3}$, then $d=-\frac{1}{4}$. If the nth term of the A. P. is $-\frac{10}{3}$, then $\frac{5}{12}-\frac{1}{4}(n-1)=-\frac{10}{3}$, whence $n=16$.

13. See Art. 263. **14.** See Art. 263, Ex. 1.

15. $b=\dfrac{2ac}{a+c}$, then $\dfrac{a}{a-b}=\dfrac{a}{a-\dfrac{2ac}{a+c}}=\dfrac{a(a+c)}{a^2-ac}=\dfrac{a+c}{a-c}$.

16. Let the numbers be $a, \dfrac{2ab}{a+b}, b$, then $\dfrac{2ab}{a+b}=12$ and $a+b=25$ so that $ab=150$. Solving $a+b=25, ab=150$, the numbers are 10, 15.

17. Let them be x and $x-8$, then $15=\dfrac{2x(x-8)}{2x-8}$ or $x^2-23x+60=0$, whence $x=20$ or 3, then $x-8=12$ or -5.

18. $b=\dfrac{2ac}{a+c}$, then $\dfrac{b+a}{b-a}+\dfrac{b+c}{b-c}=\dfrac{\dfrac{2ac}{a+c}+a}{\dfrac{2ac}{a+c}-a}+\dfrac{\dfrac{2ac}{a+c}+c}{\dfrac{2ac}{a+c}-c}=\dfrac{3ac+a^2}{ac-a^2}$ $+\dfrac{3ac+c^2}{ac-c^2}=\dfrac{3c+a}{c-a}+\dfrac{3a+c}{a-c}=\dfrac{2c-2a}{c-a}=2.$

19. $\frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{b}$ are in A. P. then $\frac{1}{b}=\frac{1}{a}+3\,d$ or $d=\frac{1}{3}\left(\frac{1}{b}-\frac{1}{a}\right)$. Then $\frac{1}{x}=\frac{1}{a}+\frac{1}{3}\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{2\,b+a}{3\,ab}$ or $x=\frac{3\,ab}{2\,b+a}$.

20. If $1+x$, $3+x$, $9+x$ are in H. P., $3+x=\frac{2(1+x)(9+x)}{10+2\,x}$.

21. $b+x=\frac{2(a+x)(c+x)}{a+c+2\,x}$. Cross multiply and solve for x.

Exercise 169 — Page 378

1–11. In the formula nth term $= ar^{n-1}$ substitute the values of a, r, and n and simplify.

12. Here $a = 18$ and $ar^3 = \frac{2}{3}$, then $r^3 = \frac{1}{27}$ or $r = \frac{1}{3}$.

13. Here $ar^2 = 40$ and $ar^4 = 160$, then $r^2 = 4$, $a = 10$.

14. If it is the nth term, then $5 \cdot 2^{n-1} = 640$ or $2^{n-1} = 128 = 2^7$, then $n - 1 = 7$ or $n = 8$.

15. $ar^2 = 1$, $ar^5 = -\frac{1}{8}$, then $r^3 = -\frac{1}{8}$ or $r = -\frac{1}{2}$, $a = 4$ and the 10th term $= ar^9 = 4 \times (-\frac{1}{2})^9 = -\frac{1}{128}$.

16. $ar^3 = .016$, $ar^6 = .000128$, then $r^3 = .008$ or $r = .2$, $a = 2$.

18. $a = 5$, $ar^3 = 135$, then $r^3 = 27$ or $r = 3$.

19. Here 64 is the 6th term, then $2\,r^5 = 64$ or $r = 2$ and the means are 4, 8, 16, 32 ; $3\,r^6 = 7\frac{1}{9}$ or $r^6 = \frac{64}{27}$, $r^2 = \frac{4}{3}$, $r = \frac{2}{\sqrt{3}}$ and the means are $\frac{6}{\sqrt{3}}$, $\frac{12}{3}$, $\frac{24}{3\sqrt{3}}$, $\frac{48}{9}$, $\frac{96}{9\sqrt{3}}$, or $2\sqrt{3}$, 4, $\frac{8\sqrt{3}}{3}$, $\frac{16}{3}$, $\frac{32\sqrt{3}}{9}$.

20. The G. mean between the 5th and 9th terms is the 7th term.

21. $\frac{1}{2}(a + b) = 10$, $\sqrt{ab} = 8$. If $a + b = 20$, $ab = 64$, $a = 16$, $b = 4$.

22. $\sqrt{ab} = b + 20 = a - 100$. Then $ab = b^2 + 40\,b + 400$, but $a = b + 120$ so $b^2 + 120\,b = b^2 + 40\,b + 400$; $b = 5$, $a = 125$.

23. $ar^2 + ar^3 = 40$, $ar^5 + ar^6 = 2560$. By division $r^3 = 64$ or $r = 4$. Then $16\,a + 64\,a = 40$ and $a = \frac{1}{2}$.

25. It is required to show that a, $\sqrt{ab}$, b, $\sqrt{bc}$, c are in G. P. The common ratio of the first three is $\frac{\sqrt{b}}{\sqrt{a}}$ and of the last three $\frac{\sqrt{c}}{\sqrt{b}}$, but these are equal since $b^2 = ac$.

26. $\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \cdots$, are in G. P., the common ratio being $\frac{1}{r}$; but the reciprocals of the terms of an A. P. are not in A. P., as $\frac{1}{a} - \frac{1}{a+d}$ is not equal to $\frac{1}{a+d} - \frac{1}{a+2\,d}$.

27. The series of rth terms $= ax^{r-1}$, ax^{2r-1}, ax^{3r-1}, $\cdots$, where x is the common ratio. These are in G. P. the common ratio being x^r.

28. If all the terms are multiplied by x, the series is $ax + axr + axr^2 + \cdots$, whose common ratio is also r.

29. Let the numbers be $\frac{a}{r}$, a, ar, then $\frac{a}{r} + a + ar = 21$ and $a^3 = 216$ or $a = 6$ whence $\frac{6}{r} + 6 + 6r = 21$ and $r = 2$ or $\frac{1}{2}$.

30. If $\frac{1}{2}(a+b) = \sqrt{ab}$, $(a+b)^2 = 4ab$ or $(a-b)^2 = 0$. Then $a = b$.

31. Let a be the first term, l the last term, and r the common ratio, then the nth term from the beginning $= ar^{n-1}$ and the nth from the end $= \frac{l}{r^{n-1}}$ and the product $= al$ which is constant for all values of n.

32. Let A be the A. mean, G the G. mean and H the H. mean, then $A = \frac{a+b}{2}$, $G = \sqrt{ab}$, $H = \frac{2ab}{a+b}$, from which $G^2 = AH$ and therefore $G : A = H : G$ and A, G, H are in G. P.

Exercise 170 — Page 382

1. $S = \frac{2^9 - 1}{2 - 1} = 511$. **2.** $S = \frac{(\frac{1}{2})^{10}}{1 - \frac{1}{2}} = 2\left(1 - \frac{1}{2^{10}}\right) = 2 - \frac{1}{2^9}$.

3. $S = \frac{1}{24} \cdot \frac{2^8 - 1}{2 - 1}$. **4.** $S = 16 \cdot \frac{1 - (-\frac{1}{2})^9}{1 - (-\frac{1}{2})} = \frac{16 + \frac{1}{2^5}}{\frac{3}{2}} = 10\frac{11}{16}$.

5. $S = \frac{(\sqrt{3})^8 - 1}{\sqrt{3} - 1} = \frac{80}{\sqrt{3} - 1}$. **6.** $S = .1 \times \frac{1 - (.1)^6}{1 - .1} = \frac{1}{9}(.999999)$.

7. $S = \frac{1 - (-2)^{2n}}{1 - (-2)}$. **8.** $\frac{1 - (-\frac{1}{2})^{2n+1}}{1 - (-\frac{1}{2})}$. **9.** $a \cdot \frac{(2a)^n - 1}{2a - 1}$.

10. $\frac{1.04^{15} - 1}{1.04 - 1} = \frac{.80094}{.04}$. **11.** $2 \cdot \frac{2^{12} - 1}{2 - 1} = 4095 \times 2 = 8190$.

12. If $a = 18$, $ar^2 = 8$, then $r^2 = \frac{4}{9}$ or $r = \pm\frac{2}{3}$ and the series are 18, 12, 8, $5\frac{1}{3}$, $3\frac{5}{9}$ and 18, -12, 8, $-5\frac{1}{3}$, $3\frac{5}{9}$.

13. The first series $= \frac{1 - x^{n+1}}{1 - x}$. The second $= \frac{1 + x^{n+1}}{1 + x}$, and their product $= \frac{1 - x^{2n+2}}{1 - x^2}$ which is the sum of the third series, since its common ratio is x^2 and number of terms is $n + 1$.

14. $\frac{16}{1 - \frac{1}{2}}$. **15.** $\frac{1}{1 - \frac{1}{3}}$. **16.** $\frac{1}{1 + \frac{1}{2}}$. **17.** $\frac{5}{1 - \frac{2}{3}}$. **18.** $\frac{3}{1 - \frac{1}{\sqrt{3}}}$.

19. $\dfrac{.5}{1-.1}$. **20.** $\dfrac{a}{1+\frac{1}{4}}$. **21.** $\dfrac{5}{1-\frac{1}{5}\sqrt{15}}$. **22.** $\dfrac{1}{1-\dfrac{1}{1.05}}$.

23. $\dfrac{a}{1-(-r)}$. **24.** $\dfrac{a}{1-r}=9$ and $a=6$, then $r=\dfrac{1}{3}$. **25.** $\dfrac{.4}{1-.1}=\dfrac{4}{9}$.

26. $.3+(.054+.00054+\cdots)=.3+\dfrac{.054}{1-.01}=\dfrac{39}{110}$.

27. $.6+.06+.006+\cdots=\dfrac{.6}{1-.1}=\dfrac{2}{3}$.

$.13+.0013+.000013+\cdots=\dfrac{.13}{1-.01}=\dfrac{13}{99}$.

$.01+(.004+.0004+\ 00004+\cdots)=.01+\dfrac{.004}{1-.1}=\dfrac{13}{900}$.

28. $S=\dfrac{1}{1-\dfrac{1}{1.025}}=\dfrac{1.025}{.025}=41$.

29. The nth term $=\dfrac{1}{2^{n-1}}$. The sum of all the terms that follow the nth term $=\dfrac{1}{2^n}+\dfrac{1}{2^{n+1}}+\cdots=\dfrac{\dfrac{1}{2^n}}{1-\frac{1}{2}}=\dfrac{1}{2^{n-1}}$.

30. Sum of 12 terms $=\dfrac{1-(\frac{1}{2})^{12}}{1-\frac{1}{2}}=2-\dfrac{1}{2^{11}}$. Sum to infinity $=2$.

31. The sums to infinity $=\dfrac{1}{1-\frac{3}{5}}=\dfrac{5}{2}$ and $\dfrac{\frac{5}{4}}{1-\frac{1}{2}}=\dfrac{5}{2}$.

32. $ar=-8$ and $\dfrac{a}{1-r}=18$, then $9r^2-9r-4=0$ from which $r=-\frac{1}{3}$ or $\frac{4}{3}$. But r must be less than 1, so that $r=-\frac{1}{3}$.

33. Since 8 is the 4th term $27r^3=8$ or $r=\frac{2}{3}$. Then $x=18$, $y=12$.

34. $r=\dfrac{2x}{5}$ and since r is less than 1, x must be less than $2\frac{1}{2}$.

35. The distance $=32+2(24+18+\cdots$ to infinity$)=32+\dfrac{48}{1-\frac{3}{4}}$.

Exercise 171 — Page 383

1. (1) This is an A. P. whose common difference is $\frac{1}{2}$. (2) This is an H. P., and the corresponding A. P. has $-\frac{1}{10}$ as common difference. (3) This is a G. P., the common ratio being $1\frac{1}{4}$.

2. If $2+x$, $8+x$, $17+x$ are in G. P., $(2+x)(17+x)=(8+x)^2$.

3. Since a, b, c are in A. P., $b-a=c-b$. The others are in A. P. since (1) $(b+x)-(a+x)=(c+x)-(b+x)$, (2) $(b-x)-(a-x)=(c-x)-(b-x)$, (3) $bx-ax=cx-bx$, (4) $\dfrac{b}{x}-\dfrac{a}{x}=\dfrac{c}{x}-\dfrac{b}{x}$.

4. The sum of n terms of this A. P. $=\frac{n}{2}\{80-7(n-1)\}=130$. Then $7n^2-87n+260=0$ or $n=5$ or $\frac{52}{7}$, but n must be an integer.

5. If they are $a-d$, a, $a+d$, then $a=9$ and $a(a-d)(a+d)=693$.

6. This is true if $(x^2-2x-1)^2+(x^2+2x-1)^2=2(x^2+1)^2$, and on simplifying this is an identity.

7. $a=27$, $a+3d=18$, then $d=-3$. If n is the number of terms, $\frac{n}{2}(54-3n+3)=117$, whence $n=6$ or 13.

8. Sum of 7 terms $=\dfrac{1-(\frac{1}{3})^7}{1-\frac{1}{3}}=\dfrac{1093}{729}$. Sum to infinity $=\frac{3}{2}$.

9. It is required to sum $101+103+105+\cdots+999$. If n is the number of terms, $101+2(n-1)=999$ or $n=450$. Then

$$S=225(101+999)=247{,}500.$$

10. $a^2b^2+b^4+a^2c^2+b^2c^2=a^2b^2+2ab^2c+b^2c^2$, then $b^4-2ab^2c+a^2c^2=0$ or $(b^2-ac)^2=0$ or $b^2=ac$.

11. If $\frac{1}{2}(a+b)=10$ and $\dfrac{2ab}{a+b}=6\frac{2}{5}$, then $ab=64$ or $\sqrt{ab}=\pm 8$.

12. If the nth term is $3n-2$, the first term is 1 and the second is 4 or the common difference is 3 and $S=\frac{n}{2}\{2+3(n-1)\}$.

13. Since the 5th term is 29, $5+4d=29$ whence $d=6$.

14. Let $b=ar$, $c=ar^2$, $d=ar^3$ then $a+b=a+ar$, $b+c=ar+ar^2$, $c+d=ar^2+ar^3$, which have the common ratio r.

15. Sum of the first $n=\frac{n}{2}\{2+2(n-1)\}=n^2$. Sum of the first $2n=4n^2$. Then the sum of the second n terms $=3n^2$.

16. The series on the right $=1\div\left(1-\dfrac{2x}{1+x^2}\right)=\dfrac{1+x^2}{1-2x+x^2}$.

17. Suppose he goes a the first day, $a+d$ the second, and so on. In 5 days he goes $5a+10d$ and in 7 days $7a+21d$. Then $5a+10d=100$ and $7a+21d=150$, whence $a=17\frac{1}{7}$, $d=1\frac{3}{7}$. If he travels 300 miles in n days, then $\frac{n}{2}\{34\frac{2}{7}+1\frac{3}{7}(n-1)\}=300$ or $n^2+23n-420=0$, then $n=12$ or -35.

18. $x=\dfrac{1}{1-a}$, $y=\dfrac{1}{1-b}$, then $\dfrac{xy}{x+y-1}=\dfrac{1}{1-ab}$, which is the sum to infinity of $1+ab+a^2b^2+\cdots$.

19. $a\cdot\dfrac{3^n-1}{3-1}=728$ or $3^n=\dfrac{1456}{a}+1$ and $a\cdot 3^{n-1}=486$ or $3^{n-1}=\dfrac{486}{a}$. But $3^n=3\cdot 3^{n-1}$, then $\dfrac{1456}{a}+1=\dfrac{1458}{a}$ or $a=2$.

20. Suppose the payment for the first year is a, then the payment for the second is $\frac{9}{10}a$ and so on. The limit of the sum of $a + \frac{9}{10}a + \cdots$ $= \dfrac{a}{1-\frac{9}{10}} = 10\,a$. Thus the limit of the sum of all the payments is 10 times the payment for the first year.

21. Let them be a, $\dfrac{2\,ab}{a+b}$, b, then the remainders are $a - \dfrac{ab}{a+b}$, $\dfrac{ab}{a+b}$, $b - \dfrac{ab}{a+b}$, or $\dfrac{a^2}{a+b}$, $\dfrac{ab}{a+b}$, $\dfrac{b^2}{a+b}$. These are in G. P., the common ratio being $\dfrac{b}{a}$.

22. The first $= a$, the last $= a + 2\,nd$, the $(n+1)$th or middle term $= a + nd$, and $a + a + 2\,nd = 2(a + nd)$.

23. $\frac{1}{12}n(n+3)$ is the sum of n terms of an A. P. Put $n = 1$ and the sum of 1 term, or the first term, $= \frac{1}{3}$. Put $n = 2$ and the sum of 2 terms $= \frac{5}{6}$, then the second term is $\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$ and the common difference $= \frac{1}{6}$.

24. Suppose there are n sides, then the sum of the angles is $(2\,n - 4)$ right angles $= 90(2\,n - 4)$ degrees. But the sum of n terms of $120 + 125 + \cdots = \dfrac{n}{2}(240 + 5\,n - 5)$. Then $n\,(235 + 5\,n) = 180(2\,n - 4)$ or $n^2 - 25\,n + 144 = 0$, from which $n = 9$ or 16. The number of sides cannot be 16, as some of the angles would be greater than 180°.

25. Let them be a, $\dfrac{2\,ab}{a+b}$, b, then since $a = \frac{1}{2}$, $\dfrac{1}{2} + \dfrac{b}{\frac{1}{2} + b} + b = \dfrac{13}{12}$, whence $24\,b^2 + 22\,b - 7 = 0$ or $b = \frac{1}{4}$ or $= -\frac{7}{6}$.

26. Sum of n natural numbers $= \frac{1}{2}n(n+1)$. If $\frac{1}{2}n(n+1) = 1000$, $n^2 + n - 2000 = 0$ or $n = \dfrac{-1 \pm \sqrt{8001}}{2} = 44.2^+$. Then the sum is first greater than 1000 when $n = 45$. The sum of the first 45 natural numbers is 1035 while the sum of 44 is 990.

27. $S_{n+2} - S_{n+1} =$ the $(n+2)$th term, $S_{n+1} - S_n =$ the $(n+1)$th term. Then $S_{n+2} - 2\,S_{n+1} + S_n = (n+2)$th term $- (n+1)$th term $= d$.

28. $S = (a + a^2 + a^3 + \cdots$ to n terms$) + (b + 2\,b + 3\,b + \cdots$ to n terms$)$
$=$ sum of a G. P. $+$ sum of an A. P.

29. $S = 1 + a + ab + a^2b + a^2b^2 + a^3b^2 + a^3b^3 + \cdots$ to $2\,n$ terms,
$= (1 + ab + a^2b^2 + \cdots$ to n terms$) + (a + a^2b + a^3b^2 + \cdots$ to n terms$)$,
$=$ sum of two geometric series.

Exercise 172 — Page 386

1. There are 5 choices for the first digit and 4 for the second, or 20 in all. They are 45, 46, 47, 48, 56, 57, 58, 67, 68, 78, and 10 others with these digits reversed.

2. There are 2 choices for each, or 2×2 choices in all.

3. There are 9 choices for the pitcher and then 8 for the catcher, or 9×8 in all.

4. There are 20 choices for the boy and 10 for the girl, or 20×10 in all.

5. There are 3 ways to go from A to B and 4 from B to C, or 3×4 ways to go from A to C.

6. There are 6 letters from which to choose. There are 6 choices for the first letter and 5 for the second, or 30 in all.

7. If 2 are chosen, there are 26 choices for the first and 25 for the second, or 26×25 for the two. If a third is chosen there are 24 choices for it, or $26 \times 25 \times 24$ in all.

8. There are 20 ways of seating the first pupil and then 19 ways for the second, or 20×19 in all.

9. There are 10 choices for the chairman, then 9 for the secretary, and then 8 for the treasurer, or $10 \times 9 \times 8$ in all.

Exercise 173 — Page 389

1. $15 \cdot 14 = 210$; $7 \cdot 6 \cdot 5 \cdot 4 = 840$; $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$.

2. $10 \cdot 9 \cdot 8 \cdot 7 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$; $\lfloor\underline{8} = 8 \cdot 7 \lfloor\underline{6} = 56 \lfloor\underline{6}$; $\lfloor\underline{n+1} - \lfloor\underline{n} = (n+1)\lfloor\underline{n} - \lfloor\underline{n} = n\lfloor\underline{n}$.

3. $\lfloor\underline{6}$ or $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 720 ways. **4.** $\lfloor\underline{7}$ or 5040 ways.

5. There are 8 different letters and the number $= {}_8P_4 = 8 \cdot 7 \cdot 6 \cdot 5$.

6. If $n(n-1)(n-2) = \frac{1}{2} \cdot 3n(3n-1)$ or $2n^2 - 15n + 7 = 0$, then $n = 7$ or $\frac{1}{2}$, but n must be a positive integer.

7. If $30n(n-1)(n-2) = (n+2)(n+1)(n)(n-1)(n-2)$, then $30 = (n+2)(n+1)$ or $n^2 + 3n - 28 = 0$, whence $n = 4$ or -7.

8. The 2 a's may be considered as 1 letter, then there are 6 different letters and the number is $\lfloor\underline{6}$ or 720.

9. The number of ways of arranging 7 things is $\lfloor\underline{7}$. If A and B must be together they may be arranged in $2\lfloor\underline{6}$ ways. (See Art. 272, Ex. 3.) Then the number of ways in which they are not together is $\lfloor\underline{7} - 2\lfloor\underline{6}$ or $5\lfloor\underline{6}$.

10. (1) The number $= {}_5P_3 = 5 \cdot 4 \cdot 3 = 60$. (2) The number $= {}_5P_1 + {}_5P_2 + {}_5P_3 + {}_5P_4 + {}_5P_5 = 5 + 20 + 60 + 120 + 120 = 325$.

11. The 5 girls may first be arranged in $\underline{|5}$ ways. Then there are 6 places (counting the two end ones) which the boys may occupy, so that the boys may be arranged in ${}_6P_5$ ways. Therefore the total number $= \underline{|5} \times 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 86{,}400$.

12. The number must have 4 digits and the first one must be 3. The other 3 digits may be chosen from the remaining 4 digits in ${}_4P_3$ or $4 \cdot 3 \cdot 2$ ways.

13. The total number is $\underline{|6}$. The number beginning with *re* is $\underline{|4}$.

Exercise 174 — Page 391

1. There are 9 choices for the first digit, 9 for the second, and 9 for the third, then the total number is 9^3 or 729.

2. $\dfrac{\underline{|7}}{\underline{|2}}, \dfrac{\underline{|7}}{\underline{|3}\,\underline{|2}}, \dfrac{\underline{|9}}{\underline{|3}\,\underline{|2}}, \dfrac{\underline{|11}}{\underline{|4}}$. (See Art. 274, Ex. 2.)

3. $\underline{|7} \div \underline{|3}\,\underline{|2}$, since there are 7 things of which three are alike and 2 are alike.

4. The only choice is in the arrangement of the 4 middle letters of which 2 are alike, then the number $= \underline{|4} \div \underline{|2}$.

5. To be even the units digit must be 2, then the other 5 may be arranged in $\underline{|5} \div \underline{|2}\,\underline{|3}$ ways.

6. The total number is $\underline{|6} \div \underline{|3}$; if it must begin with e, the other 5 may be arranged in $\underline{|5} \div \underline{|2}$ ways; if it must begin with 2 e's, the other 4 may be arranged in $\underline{|4}$ ways; if it must begin with 3 e's, the other 3 may be arranged in $\underline{|3}$ ways.

7. If we suppose the odd digits to be alike, there will be 9 things of which 5 are alike and the number $= \underline{|9} \div \underline{|5}$.

8. Here there are 9 things to be arranged of which 3 are alike, 4 are alike and 2 are alike, therefore the number of arrangements is $\underline{|9} \div \underline{|3}\,\underline{|4}\,\underline{|2}$.

9. Here there are 4 ways of disposing of the first prize, 4 for the second, 4 for the third, and 4 for the fourth, so that the total number is $4^4 = 256$.

Exercise 175 — Page 394

1. $\dfrac{\underline{|5}}{\underline{|3}\,\underline{|2}} = 10, \dfrac{\underline{|8}}{\underline{|6}\,\underline{|2}} = 28, \dfrac{\underline{|20}}{\underline{|4}\,\underline{|16}} = \dfrac{20 \cdot 19 \cdot 18 \cdot 17}{4 \cdot 3 \cdot 2 \cdot 1} = 4845.$

2. $\underline{|6} \div \underline{|3}\,\underline{|3}$, $\underline{|15} \div \underline{|12}\,\underline{|3}$, $\underline{|13} \div \underline{|5}\,\underline{|8}$, $\underline{|10} \div \underline{|7}\,\underline{|3}$.

3. The order of selection is not taken into account and this is a problem in combinations. The number $= \underline{|12} \div \underline{|3}\,\underline{|9}$.

4. ${}_{2n}C_3 = 2\,n(2\,n-1)(2\,n-2) \div \underline{|3}$, ${}_nC_3 = n(n-1)(n-2) \div \underline{|3}$.

Then $2\,n(2\,n-1)\,(2\,n-2) = 11\,n(n-1)\,(n-2)$. Removing the common factors n and $n-1$, $4(2\,n-1) = 11(n-2)$ or $n = 6$.

5. ${}_{100}C_{98} = {}_{100}C_2 = 100{\cdot}99 \div 2 = 4950$; ${}_{99}C_{97} = 99{\cdot}98 \div 2 = 4851$.

6. $(n+1)n(n-1) : n(n-1)(n-2)(n-3) \div \underline{|4} = 36 : 5$, then

$$5\,n(n+1)(n-1) = 36\,n(n-1)(n-2)(n-3) \div 24,$$

$$\therefore\ 10(n+1) = 3(n-2)(n-3) \text{ or } 3\,n^2 - 25\,n + 8 = 0 \text{ or } n = 8.$$

7. If the number of combinations 5 at a time is the same as the number 12 at a time, the number of things must be 17, so that $n = 17$. (See Art. 277.)

8. Every two points which are joined will make a line and thus the number of lines is the number of ways in which two points may be chosen from 8 and is ${}_8C_2$ or 28.

9. The total number of lines which can be formed by joining 8 points in pairs is 28, but of these 8 will be sides, thus the number of diagonals is 20. If there are n sides, the number of diagonals is ${}_nC_2 - n$.

10. Every 3 points when joined will make a triangle and will make a different triangle from that formed by joining any other three. Therefore the number is ${}_nC_3$.

11. The 5 men can be chosen from 8 in ${}_8C_5$ ways and the 5 women from 9 in ${}_9C_5$ ways. Then the 5 of each can be chosen in ${}_8C_5 \times {}_9C_5$ ways.

12. He can choose the 8 relatives from 10 in ${}_{10}C_8$ ways and the other 7 from the remaining 10 in ${}_{10}C_7$ ways. Therefore the whole choice may be made in ${}_{10}C_8] \times {}_{10}C_7$ ways.

13. The 4 consonants may be chosen in ${}_{16}C_4$ ways and the 1 vowel in 5 ways. Therefore the 5 letters may be chosen in ${}_{16}C_4 \times 5$ ways. Now each choice of 5 letters may be permuted to make $\underline{|5}$ words, therefore the number of words is ${}_{16}C_4 \times 5 \times \underline{|5}$.

14. 1 officer and 4 men may be chosen in $3 \times {}_{12}C_4 = 1485$ ways.
2 officers and 3 men may be chosen in ${}_3C_2 \times {}_{12}C_3 = 660$ ways.
3 officers and 2 men may be chosen in ${}_3C_3 \times {}_{12}C_2 = 66$ ways.
Therefore the whole choice can be made in $1485 + 660 + 66$ ways.

15. ${}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4 + {}_5C_5 = 5 + 10 + 10 + 5 + 1 = 31$.

16. It is required to show $\dfrac{\underline{|n+1}}{\underline{|r}\,\underline{|n-r+1}} = \dfrac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} + \dfrac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}}$.

Divide each numerator by $\underline{|n}$ and each denominator by $\underline{|r-1}\,\underline{|n-r}$ and it remains to show that $\dfrac{n+1}{r(n-r+1)} = \dfrac{1}{r} + \dfrac{1}{n-r+1}$, which is easily seen.

17. (1) Since A must be chosen we have to choose 5 others from the remaining 14 and this may be done in ${}_{14}C_5$ ways.

(2) Since A must not be chosen we must choose 6 from the remaining 14 and this may be done in ${}_{14}C_6$ ways.

Exercise 176 — Page 396

1. (1) The first prize may be disposed of in 20 ways, the second in 19, and the third in 18 or the three prizes in $20 \cdot 19 \cdot 18$ ways. (2) Each may be disposed of in 20 ways or the three in $20 \cdot 20 \cdot 20$ ways.

2. The total number is $\lfloor\underline{n}$. The number in which A and B are together is $2\lfloor\underline{n-1}$, (see Art. 272, Ex. 3), then the number in which they are not together is $\lfloor\underline{n} - 2\lfloor\underline{n-1}$.

4. (1) This is the same as the number of ways that they may be arranged in a line and is $\lfloor\underline{8}$. (2) Here the first person has no choice, but when he is in position the other 7 may be arranged in $\lfloor\underline{7}$ ways.

5. (1) The number is ${}_9P_4 = 9 \cdot 8 \cdot 7 \cdot 6$. (2) Here the number is 9^4, since there are 9 choices for each digit.

6. For each station 24 different tickets are required, or 24×25 in all.

7. If 1 only is chosen the number is 6, if 2 are chosen it is 6^2 and so on. The total number $= 6 + 6^2 + 6^3 + 6^4 = 1554$.

8. Of the first word there are $\lfloor\underline{6} \div \lfloor\underline{3}$ or 120, and of the second there are $\lfloor\underline{5} \div \lfloor\underline{2} = 60$.

9. To be greater than a million all the digits must be used and this may be done in $\lfloor\underline{7}$ ways, $\frac{1}{7}$ of these will begin with 0 and will be less than a million, then the number greater than a million is $\frac{6}{7}\lfloor\underline{7}$.

10. Every two lines chosen will give one point of intersection and therefore the number of points is ${}_nC_2 = \frac{1}{2}n(n-1)$.

11. It is greatest when r is nearest to $\frac{1}{2}$ of the whole number of things which in (1) is n and in (2) is n or $n+1$.

12. A polygon which has n side has $\frac{1}{2}n(n-3)$ diagonals. (See Ex. 9, page 394.) If $\frac{1}{2}n(n-3) = 44$, $n = 11$ or -8.

13. 1 master and 4 pupils may be chosen in $3 \times {}_{10}C_4 = 630$ ways.
2 masters and 3 pupils may be chosen in ${}_3C_2 \times {}_{10}C_3 = 360$ ways.
3 masters and 2 pupils may be chosen in ${}_3C_3 \times {}_{10}C_2 = 45$ ways.

Therefore the total number is 1035.

14. This is the same as the number of ways in which one boy may receive 5 of the books and is ${}_{10}C_5$.

15. A may get 2 in ${}_9C_2$ ways. There are now 7 left and B may get 3 of these in ${}_7C_3$ ways, then the other 4 are given to C. The total number $= {}_9C_2 \times {}_7C_3 \times 1 = 1260$.

16. There is only 1 way of choosing the goal keeper. The other 6 may be chosen from 11 in ${}_{11}C_6$ ways.

17. The first player may receive 13 cards in ${}_{52}C_{13}$ ways. The second can receive 13 from the remaining 39 in ${}_{39}C_{13}$ ways, then the third can receive 13 in ${}_{26}C_{13}$ ways, then the fourth receives the remaining 13 which can be done in only 1 way. The number of ways of making the distribution is ${}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times 1$.

18. 3 Con. and 2 Lib. may be chosen in ${}_7C_3 \times {}_4C_2 = 210$ ways.
4 Con. and 1 Lib. may be chosen in ${}_7C_4 \times {}_4C_1 = 140$ ways.
5 Con. and 0 Lib. may be chosen in ${}_7C_5 \qquad = \ 21$ ways.

The total number of ways in which it can be done $= 371$.

19. There is a choice of only 5 balls from the remaining 12 and therefore the number of choices is ${}_{12}C_5$.

20. The total number $= \underline{|3n} \div \underline{|n}\,\underline{|2n}$. The number in which a particular thing is chosen is ${}_{3n-1}C_{n-1} = \underline{|3n-1} \div \underline{|n-1}\,\underline{|2n}$ and this is $\frac{1}{3}$ of the total number since $\underline{|3n} = 3n\,\underline{|3n-1}$ and $\underline{|n} = n\,\underline{|n-1}$.

21. There are 3 choices in posting each of the 5 letters so that the number of ways $= 3^5 = 243$.

22. The 3 men may be chosen in ${}_8C_3$ or 56 ways. The 2 women may be chosen in ${}_5C_2$ or 10 ways. The 3 men may now be assigned to the 3 positions in $\underline{|3}$ or 6 ways and the 2 women in 2 ways. Thus the total number $= 56 \cdot 10 \cdot 6 \cdot 2 = 6720$. If the 2 women or the 3 men may hold any of the 5 positions, the result would be $56 \cdot 10\,\underline{|5} = 67{,}200$.

23. There will be $\underline{|5}$ or 120 numbers in all. Suppose they were written down in a column for addition. Then the units column would contain each digit 24 times and therefore the sum of the units is $24(1 + 2 + 3 + 4 + 5)$ or 360. The sum of each of the other columns would evidently be the same. Thus the total sum $= 360 + 3600 + 36{,}000 + 360{,}000 + 3{,}600{,}000 = 3{,}999{,}960$.

24. The greatest number of combinations of $2n$ things is given by taking them n at a time and the number is $\underline{|2n} \div \underline{|n}\,\underline{|n}$. The greatest number for $2n - 1$ things is given when they are taken n or $n - 1$ at a time and this number is $\underline{|2n-1} \div \underline{|n}\,\underline{|n-1}$. This latter number is $\frac{1}{2}$ of the former since $\underline{|2n} = 2n\,\underline{|2n-1}$ and $\underline{|n} = n\,\underline{|n-1}$.

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